LIBRARY COPY

Institute of Polar Studies

Report No. 82

Glacial Movements in Greenland from Doppler Satellite Observations

by

Alice Jean Remington Drew

Institute of Polar Studies
and
Department of Geodetic Science
and Surveying

1983

OSU

The Ohio State University
Institute of Polar Studies
Columbus, Ohio 43210

INSTITUTE OF POLAR STUDIES

Report No. 82

GLACIAL MOVEMENTS IN GREENLAND FROM DOPPLER SATELLITE OBSERVATIONS

by

Alice Jean Remington Drew

Institute of Polar Studies
and
Department of Geodetic Science and Surveying
The Ohio State University

1983

Institute of Polar Studies
The Ohio State University
Columbus, Ohio 43210

This research was financed in part by

NSF grant DPP-8008356A01
and
NASA Research grant NSG5265

Copies of this and other publications of the Institute of Polar Studies are available from:

Institute of Polar Studies
The Ohio State University
125 South Oval Mall
Columbus, Ohio 43210
Telephone: 614-422-6531

ABSTRACT

Seven Magnavox MX1502 satellite receivers were used during the summers of 1980 and 1981 to obtain the movement of 22 stations at three locations on the ice sheet of interior Greenland. The research program was successful with only minor equipment problems. Severe weather conditions that delayed the servicing of the receivers and operator errors resulted in the loss of only a minor amount of data. Two receivers were located at stationary sites on the west coast of Greenland for the whole of both seasons. This allowed the short arc method to be used to obtain relative coordinates with higher precisions than are obtainable by point positioning. The stations on the ice sheet were located at an approximate latitude of 65 degrees North and crossed the southern dome of Greenland. The positions of the stations on the ice sheet were obtained with formal accuracies of better than 0.20 meters. However, the actual accuracies obtained were not this precise, particularly in the elevations. The station coordinates were obtained from the satellite data. From the coordinates, the station velocities, as well as ice sheet slopes, and the baseline lengths between the stations were calculated. Changes in the baseline lengths between 1980 and 1981 were used to calculate strain rates.

The two stations that are nearest to the ice crest are not moving in the expected direction (northeast) but instead are moving in a direction slightly west of north. This indicates that the positions of the ice crest and the ice divide do not coincide. The other stations west of the ice divide are moving 50 to 75 degrees west of north. The stations east of the ice divide are moving 50 to 70 degrees east of north. The stations farther from the ice divide are moving more nearly east or west. The angle between the direction of maximum extension and the direction of motion is generally small with the direction of maximum extension more nearly east-west. Thus, there is no major surface shearing in the motion of the ice on either side of the ice divide. In two of the three areas studied, the minimum strain is compressive. The magnitude of the maximum strain and the velocities increase away from the ice divide and with increasing slope.

Acknowledgements

The research presented in this report was partially financed by the National Science Foundation grant DPP.8008356A01 to Ian M. Whillans, Institute of Polar Studies, The Ohio State University. Partial financial support was also provided by the NASA Research Grant No. NSG 5265 to Ivan I. Mueller, Department of Geodetic Science and Surveying, The Ohio State University. Computer support was provided by the Instruction and Research Computer Center of The Ohio State University.

I would like to thank Dr. Ivan I. Mueller for his continued assistance and guidance during the course of this research. I would also like to thank Dr. Ian Whillans for his advice and encouragement and for inviting me to be a member of the field crew during the second field season. All of the many people from The Ohio State University and other places who worked in Greenland in 1980 and 1981 operating the satellite receivers and collecting the other data deserve many thanks. I would like to thank Brent Archinal for his help with the computer program GEODOP and all the other students in the Geodetic Science and Surveying Department who helped and encouraged me in this endeavour. Finally, I would like to give special thanks to my husband, Richard Drew, for his continual support.

This report was presented as a thesis to the Graduate School of The Ohio State University as partial fulfillment of the Requirements for the Degree of Master of Science.

CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

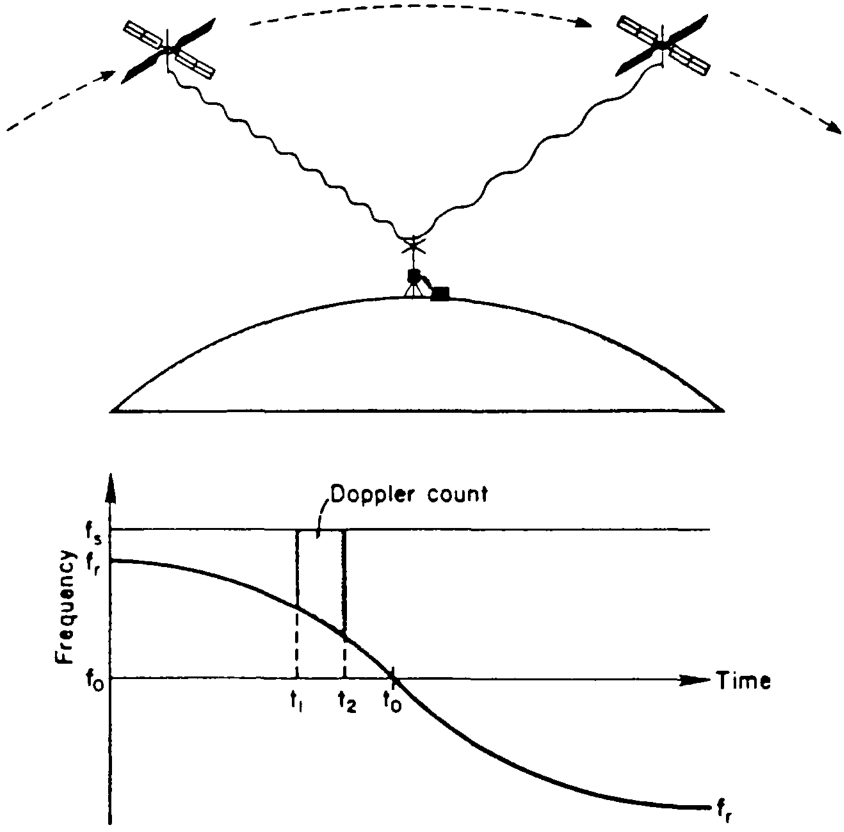
INTRODUCTION AND BACKGROUND

Surveying with Doppler Satellite Receivers

The Doppler effect is the change in the frequency of a signal from a transmitter that is moving relative to the receiver. A familiar example of this effect is the change in the pitch of a train whistle as it passes. Electromagnetic signals also show this effect. In particular, a signal from a transmitter on a satellite will appear to change frequency when received by a receiver on the ground (see Figure 1). At the receiver a reference frequency is subtracted from the signal, to give a "beat frequency." This beat frequency is then electronically integrated over short, precisely timed intervals. This is termed the "Doppler count" and is an indirect measure of the change in the distance between the satellite and the satellite receiver during that time interval. One satellite pass generally provides ten to twenty Doppler counts. Thus many range differences are obtained from each satellite pass.

The range differences calculated from a satellite pass can be used to correct a previously calculated range to the satellite. This range must be corrected for systematic effects such as tropospheric and ionospheric propagation delays and the systematic errors inherent in the electronics of the satellite receiver. A first-order ionospheric correction can be calculated if the signal is received on two different carrier wave frequencies. The errors inherent in the satellite receiver can be determined by calibrating the receiver and the antenna. Meteorological data can be used to minimize the tropospheric effect.

The Navy Navigational Satellite System (NNSS) was used in this research effort to obtain position data with high geodetic accuracy. This system is also known as the TRANSIT system. There are six satellites in this system, but only four or five are normally in operation at any one time. The Navy maintains a system of ground stations which make daily observations of these satellites. With these observations, the orbits of the satellites are predicted and the orbital parameters are injected periodically into the satellite's memory. This orbital information is the



f_r = Received frequency

f_0 = Frequency transmitted by satellite

f_s = Reference frequency

t_0 = Time of closest approach

Illustration 1: The Doppler Effect

broadcast ephemeris which is transmitted by the satellites. It is broadcast on two carrier frequencies along with very precise two-minute time signals. The orbital parameters in the broadcast ephemeris provide an orbit correct to within 20 to 30 meters. For several of the satellites, a precise ephemeris is also available. This is available several weeks after the data for it have been collected and is correct to within two meters.

There are several Doppler satellite receivers available commercially. The receivers used in this experiment were MX1502's manufactured by Magnavox. The error associated with the electronics of this type of receiver was calculated by Magnavox and is considered by them to be the same for all receivers. However, other research has shown that the oscillators in these receivers can have a very high drift rate which can significantly affect the coordinates (Schenke, 1982). The phase center of the receiver antenna is the position to which the receiver location actually refers. It is marked by Magnavox with a painted band. Like the receivers, the antennas are not individually calibrated.

Using the orbital information broadcast by the satellite, the Doppler counts (and hence, range differences between satellite and receiver) and iteration from an approximate location for the receiver, the two-dimensional position of the receiver is calculated for each pass. With two or more two-dimensional positions, a three-dimensional position is obtained. The phase center of the antenna is the position to which the calculated coordinates would refer if there were no systematic or random errors. These positions are calculated by a microprocessor in the receiver immediatly after each pass. After approximately 30 passes, the position of the station is refined to within 10 meters. These are "point positions" because the coordinates for each station are determined separately and the satellite orbits are assumed to be known.

The data received by the MX1502 are also recorded on cassette tapes. After the field season, these data may be post-processed to obtain more accurate positions for the receiver stations. Either the broadcast or the precise ephemeris may be used in post-processing. There are several techniques for obtaining the station coordinates. The point positioning method is essentially the same as described above. However, other information, such as meteorological data, may be used to improve the solution. Using the precise ephemeris increases the accuracy of the solution, even though fewer passes are normally available.

The other technique used in this report is the short arc method (Brown, 1976). Here, while the shape of the satellite orbit is considered known, its position in space is not. The coordinate system is thus not fixed by the coordinate system of the satellites, but is instead fixed by including satellite observations from known or defined stations (see Figure 2). The satellite observations from the known positions must be made simultaneously with the observations from the stations whose positions are being sought. Thus, a minimum of two receivers must be operating simultaneously. However, to fully define a coordinate system, six coordinates distributed over three stations should be known. Fixing only the three position coordinates of one station allows no translations of the coordinate system, but still allows three rotations. Setting the three coordinates of one station and two coordinates of a second station to known values still allows one rotation. If all six coordinates of the two stations were fixed, an unwarranted constraint would be placed on the scale of the coordinate system, while a rotation around the axis connecting the two staions would still be allowed. Positions determined with the short arc method are all relative with respect to the stations whose coordinates are used in the definition.

Greenland Ice Sheet Program (GISP)

This project is part of the Greenland Ice Sheet Program (GISP), a joint U.S.-Danish-Swiss scientific and engineering research program created to investigate the surface, inner structure and subsurface character of the Greenland ice sheet. To this end, Doppler satellite observations were obtained during the summers of 1980 and 1981. Seven Magnavox MX1502 receivers were used. Two were kept throughout all observations at stationary locations on the west coast of Greenland while the remaining five receivers were used for measurements on the Greenland ice sheet at twenty-two stations. The ice sheet strain information calculated using the changes in station locations between the 1980 and 1981 field seasons will be used by other invesitgators to better refine the dynamics of ice sheet movement.

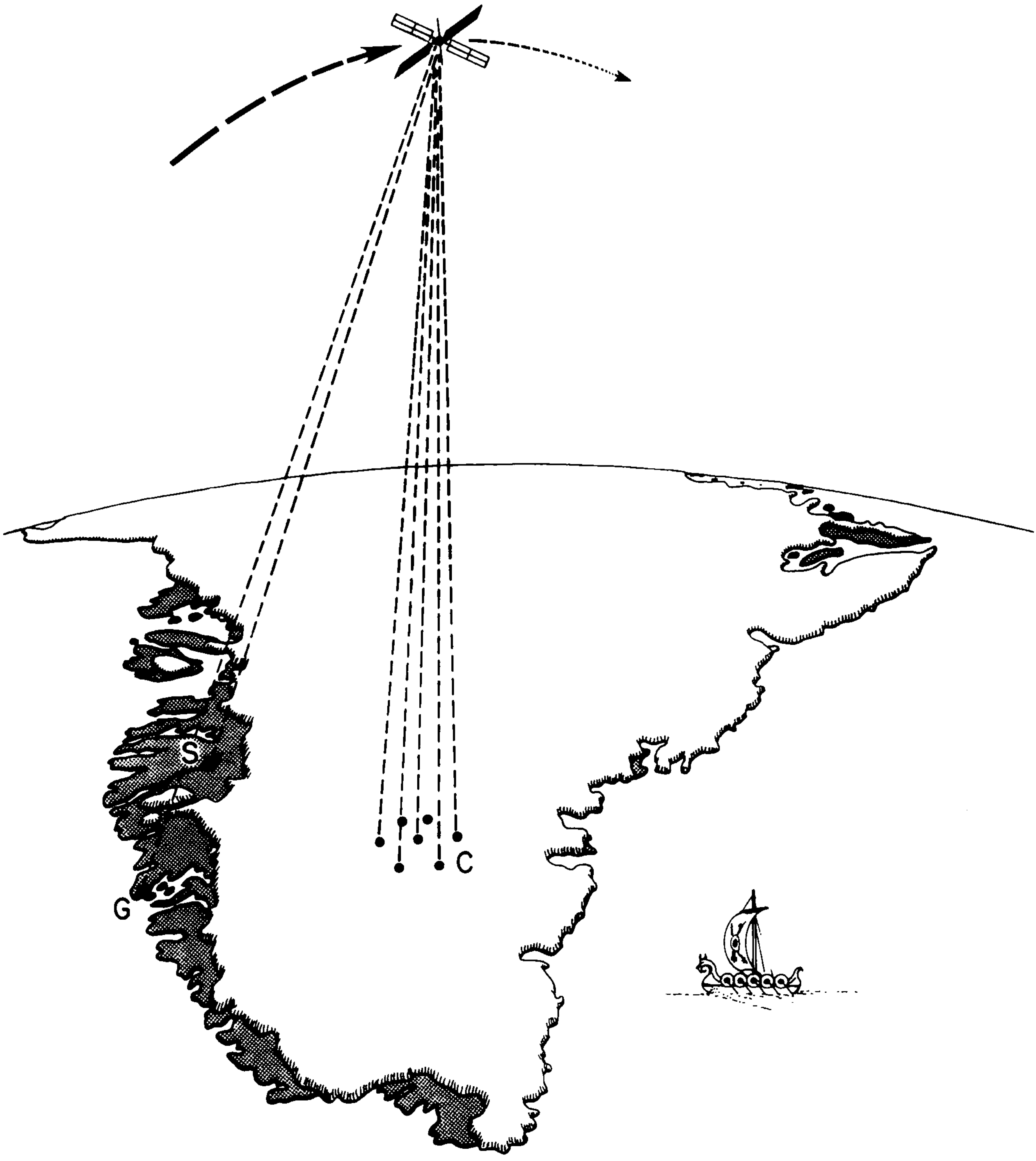


Illustration 2: The Short Arc Method

Figure 3 shows the station locations. The stations are located at approximately 65 degrees north latitude crossing the southern dome of the Greenland ice sheet and are between longitudes of 311.9 and 316.5 degrees east. The stations comprise three large strain figures. The farthest west (and at the lowest elevation) is referred to as the "Western Cluster." The stations in this group are numbered 1001 through 1007. The middle strain figure is called "Central Cluster." Its stations are numbered 2001 through 2007. These two figures are hexagons, and are on the west side of the ice crest. The easternmost strain figure is approximately rectangular. It is called the "Eastern Cluster." Its stations, numbered 3001 through 3008, are on the east side of the ice crest and this figure has within it the site of deep drilling at DYE-3 (near station 3003). One of the two coast stations was located at Godthaab, the capital of Greenland. This station is abbreviated as GOT in the tables and figures. The other station was located at Søndrestrøm Fjord. It is abbreviated as SFJ in the tables and figures.

Previous Deformation Studies of the Greenland Ice Sheet

Several geodetic surveys were made across Greenland by the International Greenland Glaciological Expedition (EGIG). These surveys were in central Greenland across the main dome of the ice sheet. The surveys made by EGIG were all north of the area covered in this experiment. In 1959 EGIG leveled a profile of nearly 700 km (Malzer, 1968). In 1968 this line was releveled. The elevations at the center of the ice sheets increased by 0.5 meters over the nine years for an average change of 0.06 m/yr (Seckel and Stober, 1968).

In 1959 and in 1967 EGIG surveyed a 651 km long chain of 77 quadrilaterals (Hofmann, 1974). The surveying was done with tellurometers. When both of the diagonal distances across a quadrilateral could not be measured, angles were obtained with a theodolite to provide some redundancy in the calculations. The chain of quadrilaterals was tied to mean sea level and to fixed stations located in the mountains on either side of Greenland. Unfortunately, the coordinates of only one fixed station were known, so the results lack sufficient control. From the data obtained in both surveys, velocity vectors were determined (Hofmann,

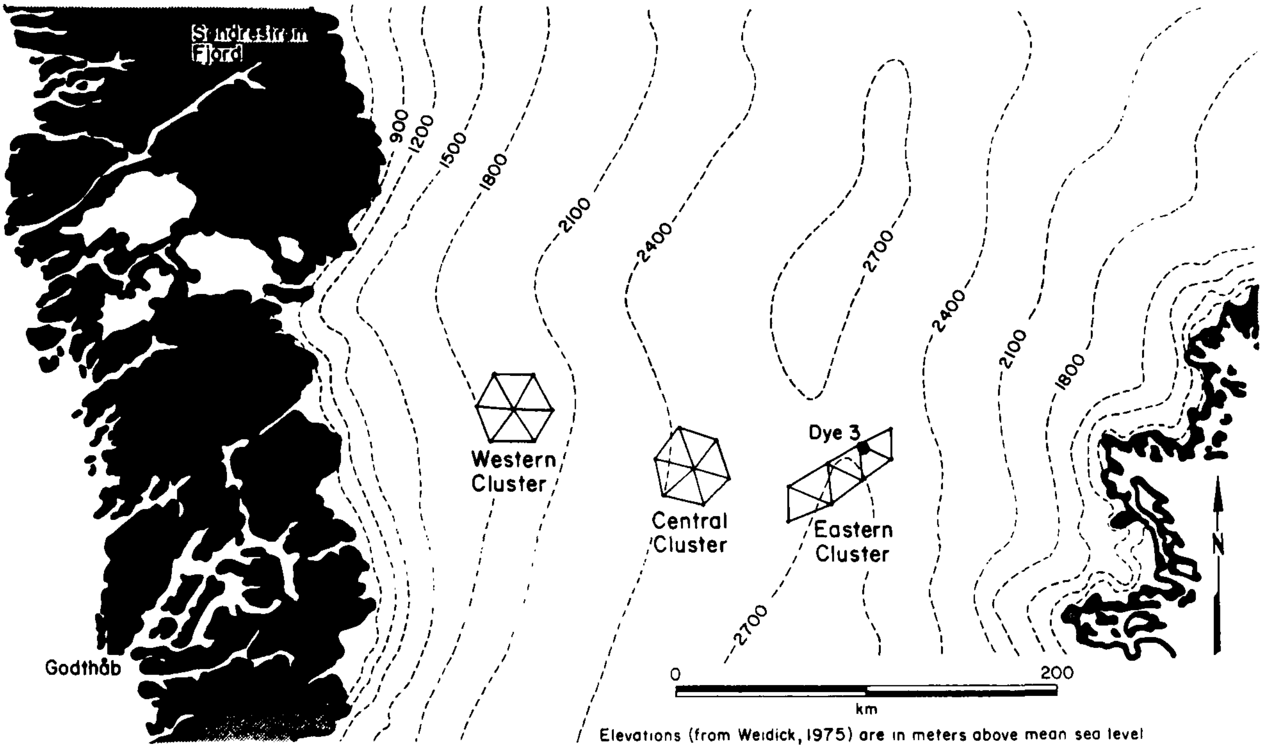


Illustration 3: Greenland Ice Sheet Stations

1974). Using estimates of the ice thickness and accumulation rate as well as computed velocities, Mock calculated a change in the thickness of the ice sheet of -0.37 m/yr (Mock,1976).

During the 1974 EGIG expedition, a deformation polygon in central Greenland, originally surveyed in 1968, was remeasured (Karsten and Stober, 1975). The strain rates calculated were 1150 $\pm$ 30 parts per million per year for the maximum strain rate and 110 $\pm$ 30 parts per million per year for the minimum strain rate. The azimuth of the maximum strain was 72.2 $\pm$ 1.3 degrees (Stober, 1976). Using the data of Karsten and Stober, and making several assumptions, Mock calculated the change in thickness of the ice sheet at this location to be -0.25 m/yr (Mock, 1976).

GISP established eight Geoceiver stations during the summers of 1971-1975 (Mock, 1976). These stations were all located on or near the crest of the ice sheet. One station (at Dye-3) was occupied three times. This station is about 2 km from one of the stations in this experiment (station 3003 in the Eastern Cluster). The two velocities Mock obtained for this station were consistent. They show a motion of about 12.7 m/yr at an azimuth of 61 degrees (Mock 1976).

Description of the Field Data Collection

Station occupation. The clusters were occupied during 1980 and 1981. At the Western and Central Clusters, receivers were placed at three stations for the entire stay at the cluster. The remainimg two receivers were placed at two of the remaining four stations for approximately half the stay. They were then moved to the other two sites. At the Eastern Cluster, receivers were placed at two stations for the entire stay while the remaining three receivers were moved about halfway through the occupation. Table 1 gives the dates of occupation and the number of passes used from each station. The amount of down time for the receivers (due to instrument failure and to operator error) was generally small, but, for some stations, amounted to several days. This down time accounts for most of the variability in the number of passes per day (in Table 1) within a given year. The change in the number of operating satellites (from five in 1980 to four in 1981) accounts for the yearly change in the number of passes per day accepted by the receiver. Note that the ratio of the number of accepted passes per day is not the same as the ratio of the operating satellites. This is because with more satellites, there is a higher probability of overlapping passes. Because the receiver can observe only one pass at a time, the second of the two overlapping passes is not recorded.

Recovery of antenna locations. Because the phase center of the antenna is the position to which the station coordinates refer, it was necessary to obtain the offset of the position of the antenna during the occupation of a station in the second year with respect to its position during the first year (see Figure 4). The antenna offset is determined relative to a datum placed in the firn below the depth of most firn compaction. Firn is the intermediate stage between snow and ice. The antenna offset does not include any position changes due to the motion of the ice, but does correct for the sinking due to accumulation of more snow and the compaction of snow into ice. The antennas were not calibrated as part of this experiment. Instead it was assumed that the band painted on the antenna was, in fact, its phase center.

While each station was occupied on the ice sheet, two or more datums were placed in the ice approximately 10 meters from each antenna. A datum consisted of a wire, one end of which was frozen to the bottom of a deep, one inch diameter hole in the ice. The depths of the holes varied from 10 to 36 meters, but most were either 15 or 20 meters deep. This is below the depth at which it is believed that most of the compaction of the snow into ice occurs. A knot was tied in the free end of the wire. Then the difference in height between the knot and the band painted on the antenna was determined by leveling.

When the antenna was removed from the site, three bamboo poles taped together were placed in the hole in the firn in which the antenna had stood. All these poles were found in place the second summer and all but two were close to upright. One pole (at station 1006) was found lying almost on its side. The antenna at this location was placed where it was believed that the taped poles had been located. This position was later comfirmed to be correct to within a few centimeters. Another pole (at station 1005) was tipped slightly north-northwest. The horizontal displacement of

TABLE 1

Observation Statistics

STA.	1980 (5 NNSS SATELLITES) DATE:HOUR[1]	PASSES[2]	PASSES /DAY	1981 (4 NNSS SATELLITES) DATE:HOUR[1]	PASSES[2]	PASSES /DAY
WESTERN CLUSTER SOLUTIONS						
1001	171:20-183:10	325	28.1	161:10-178:18	368	21.2
1002	171:21-183:10	264	22.9	161:21-178:11	346	20.9
1003	171:17-179:15	197	24.9	161:23-168:19	174	25.5
1004	171:17-178:12	183	26.9	161:19-168:15	183	26.8
1005	171:21-183:19	340	28.5	161:18-177:13	361	22.9
1006	179:19-182:00	53	24.0	168:19-176:00	134	21.0
1007	179:19-183:19	102	25.5	168:23-178:12	207	21.7
GOT	173:01-184:12	243	22.0	161:10-182:22	440	20.5
SFJ	171:17-184:12	282	21.2	161:10-182:22	437	20.3
CENTRAL CLUSTER SOLUTIONS						
2001	185:18-196:20	286	25.8	184:13-201:16	314	18.3
2002	185:16-197:15	322	26.9	184:14-201:22	161	9.3
2003	186:19-191:23	139	26.9	185:13-195:14	199	19.8
2004	185:20-191:15	171	29.5	184:01-195:14	223	19.3
2005	185:20-196:21	261	23.6	184:01-201:16	384	21.8
2006	192:21-197:12	116	25.1	195:17-201:09	156	27.5
2007	192:19-197:13	128	26.9	195:16-201:18	150	24.7
GOT	184:12-198:12	308	22.0	183:00-197:07	333	22.7
SFJ	184:12-198:12	308	22.0	183:00-201:22	434	22.9
EASTERN CLUSTER SOLUTIONS						
3001	199:14-212:12	284	22.0	203:16-207:16	89	22.3
3002	199:15-212:14	315	24.3	205:15-208:11	73	25.8
3003	199:15-212:11	342	26.6	203:17-208:15	129	26.2
3004	199:14-217:21	443	24.2	203:14-215:10	233	19.7
3005	199:17-217:18	446	24.7	203:12-215:12	273	22.8
3006	212:20-217:16	130	26.9	208:17-215:10	87	13.0
3007	212:18-217:21	145	28.3	209:21-215:12	137	24.4
3008	212:14-217:21	151	28.5	209:18-215:10	116	20.5
GOT	198:13-217:09	432	22.9	208:08-215:08	148	21.1
SFJ	198:13-217:18	426	22.2	203:12-215:12	201	16.8

[1] DATE:HOUR - From the first pass used in the solution to the last pass used. Dates are numbered from January 1.

[2] PASSES - The number of satellite passes observed at that station and used in that solution. (Each cluster, each year is a separate solution.)

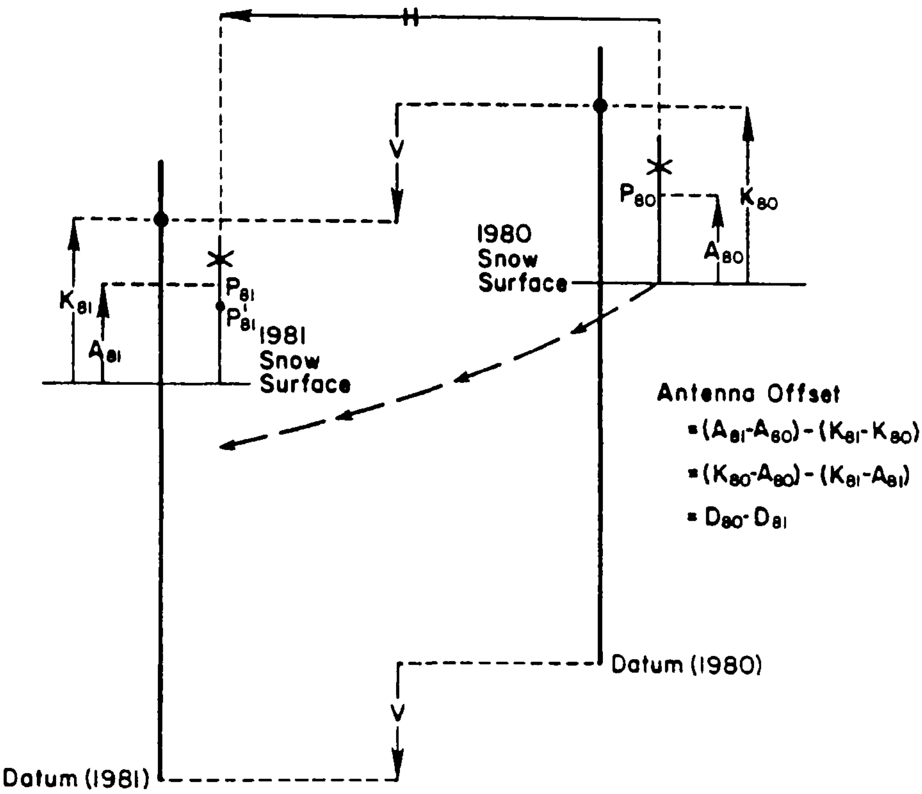


Illustration 4: Vertical Velocity of Antenna With Respect To The Ice Datum

A = Height of phase center of the antenna above the snow surface
K = Height of knot above the snow surface (at the antenna)
D = K-A

P = Phase center of antenna
P'_{81} = Position of phase center in 1980 after downslope motion (with respect to the datum)
H = Horizontal movement
V = Vertical movement

Coordinates listed in the solutions are for positions P_{80} (1980) and P'_{81} (1981).

the antenna was not accuratly measured but is believed to be under 0.10 meters. At all the other locations, the antennas were placed right next to the taped poles (which had frozen in place). Thus, the antennas were displaced horizontally perhaps two to three centimeters, usually to the south.

While the stations were occupied the second year, the datum wires were dug out of the new snow. At least one datum was recovered for each receiver location. The difference in height between the knot and the band painted on the antenna was again determined by leveling. From the leveling differences of both years, the vertical offset of the antenna was calculated. The measurements and calculated antenna offsets for each datum are given in Table 2. The elevations listed are heights above the snow surface at the antenna (assuming that the staff did, in fact, rest on the snow).

If more than one datum was recovered for a station, the antenna offsets calculated for each datum were averaged. In two cases, antennas were moved (vertically only) during the 1981 occupation. These stations were releveled and the antenna offsets from both levelings were averaged. The antenna offsets listed in Table 13 (p. 45) are the offsets used in the calculations of the coordinates. In two cases (at stations 1002 and 2001) there are differences of these offsets with the average offsets listed in Table 2 due to blunders in their calculation. These errors are only a few centimeters and smaller than the standard deviations of the elevations.

At thirteen of the stations, the antenna offsets calculated from the two or three datums at each station differ by less than 0.02 meters. This is an acceptable variation due to variations in the compaction rates. At four more stations, the antenna offsets calculated for the station differ by 0.05 to 0.08 meters. These variations are large, but not significant as they are less than the formal standard deviations of the elevations. The antenna offsets calculated for station 1002 vary by 0.21 meters. At this station the depths of the datums range from 5 to 36 meters. However, the antenna offsets calculated from the two datums nearest in depths to the datums at other stations (10 and 15 meters) differ by only 0.01 meters. The two antenna offsets calculated for station 2004 differ by 0.16 meters. It is possible that one of these two

TABLE 2

Antenna Offsets

STA.	DATUM #	DEPTH[1]	ELEV. 1980	ELEV. 1981	CHANGE (81-80)	ANTENNA OFFSETS
			(m)	(m)	(m)	(m)
1001	A		0.98	1.18	0.20	
	1		1.94	1.13	-0.82	1.02
	2		1.97	1.15	-0.82	1.02
	3		1.94	1.11	-0.83	1.04
						1.02
1002	A		1.10	1.12	0.02	
	2	10	2.31	1.44	-0.86	0.88
	3	10	2.26	1.38	-0.88	0.90
	A		1.10	1.01	-0.09	
	2	10	2.31	1.25	-1.05	0.96
	3	10	2.26	1.18	-1.07	0.98
						0.93
1003	A		0.99	1.20	0.21	
	1		2.51	1.62	-0.90	1.10
	2		2.28	1.38	-0.90	1.10
	3		1.80	0.85	-0.95	1.15
						1.12
1004	A		1.49	1.04	-0.46	
	1		2.39	1.52	-0.87	0.41
	2		1.83	0.95	-0.88	0.42
	3		2.28	1.42	-0.86	0.40
						0.41
1005	A		1.18	0.94	-0.24	
	2		2.13	1.01	-1.12	0.88
	3		2.21	1.08	-1.14	0.90
						0.89
1006	A		1.57	0.93	-0.64	
	1		2.13	1.47	-0.67	0.02
	2		2.17	1.47	-0.70	0.06
	3		2.03	1.29	-0.74	0.10
						0.06
1007	A		1.17	1.13	-0.03	
	1		2.19	1.25	-0.94	0.91
	2		2.37	1.43	-0.94	0.91
	3		2.32	1.37	-0.94	0.91
						0.91

TABLE 2 - CONTINUED

STA.	DATUM #	DEPTH[1]	ELEV. 1980	ELEV. 1981	CHANGE (81-80)	ANTENNA OFFSETS
			(m)	(m)	(m)	(m)
2001	A		1.02	0.90	-0.12	
	2	15	2.05	1.44	-0.61	0.49
	A		1.02	0.66	-0.36	
	2	15	2.05	1.34	-0.71	0.35
						0.42
2002	A		1.09	1.20	0.11	
	1	5	1.57	1.18	-0.39	0.49
	2	10	2.10	1.80	-0.29	0.40
	4	36	2.09	1.91	-0.17	0.28
	5	15	2.11	1.83	-0.28	0.39
						0.39
2003	A		1.13	1.07	-0.06	
	2		1.99	1.31	-0.68	0.62
						0.62
2004	A		1.19	1.36	0.17	
	1		2.05	1.45	-0.60	0.77
	2		2.14	1.38	-0.76	0.93
						0.85
2005	A		1.13	1.12	-0.00	
	1	20	2.05	1.45	-0.60	0.60
	2	20	2.04	1.43	-0.61	0.60
						0.60
2006	A		1.22	1.03	-0.18	
	1	15	2.12	1.50	-0.62	0.44
	2	15	2.18	1.43	-0.61	0.44
						0.44
2007	A		1.17	0.96	-0.21	
	1	15	2.27	1.72	-0.55	0.34
	2	15	2.16	1.62	-0.54	0.33
						0.33
3001	A		1.00	1.03	0.03	
	1		1.71	1.27	-0.44	0.47
	2		1.77	1.25	-0.51	0.54
	3		1.82	1.35	-0.48	0.51
						0.51
3002	A		0.85	1.00	0.15	
	1		1.79	1.34	-0.45	0.60
	2		1.94	1.48	-0.46	0.60
	3		2.01	1.55	-0.46	0.60
						0.60

TABLE 2 - CONTINUED

STA.	DATUM #	DEPTH[1]	ELEV. 1980	ELEV. 1981	CHANGE (81-80)	ANTENNA OFFSETS
			(m)	(m)	(m)	(m)
3003	A		1.24	0.96	-0.28	
	1		2.12	1.42	-0.70	0.42
	2		2.21	1.51	-0.70	0.42
	3		2.23	1.55	-0.68	0.40
						0.41
3004	A		0.92	1.03	0.11	
	1	15	2.07	1.80	-0.27	0.37
	2	20	2.18	1.93	-0.25	0.36
	3	15	2.24	1.99	-0.25	0.36
						0.36
3005	A		0.87	0.75	-0.12	
	1		1.82	1.47	-0.35	0.22
	2		1.74	1.42	-0.32	0.20
	3		1.92	1.58	-0.34	0.22
						0.21
3006	A		1.20	1.04	-0.16	
	1	16	2.52	2.10	-0.43	0.27
	2	17	2.54	2.12	-0.42	0.26
						0.27
3007	A		1.18	0.90	-0.29	
	1		2.11	1.32	-0.79	0.51
	2		2.14	1.35	-0.80	0.51
						0.51
3008	A		1.15	0.89	-0.25	
	1	15	2.17	1.60	-0.57	0.31
	2	15	2.13	1.64	-0.49	0.23
						0.27

[1] Depths not listed were at standard depths of 15 to 20 meters.

datums was not, in fact, frozen in place. It may later have slipped, causing the discrepancy of 0.16 meters.

Due to the inexperience of the leveling crew, several blunders were made in obtaining and recording the leveling data. These blunders have been corrected whenever possible. It is still possible that blunders remain in the antenna offsets. If so, they will probably be height differences in integer multiples of the units of the leveling rod (1.0 foot or approximately 0.30 meters). From the field notes, it appears possible that the height of the antenna at station 2007 above the snow surface may be off by one foot. Because of these problems, the accuracy of the antenna offsets is uncertain.

Expected Results

Direction of motion. An ice crest is the line of maximum elevations in an ice sheet. It corresponds to a continental divide. An ice divide is the line from which the ice flow diverges. Theoretically, ice must move in the direction of maximum slope. Thus, the ice crest and the ice divide should coincide. The exact position and azimuth of the ice crest in Southern Greenland is not known. From the maps available it was thought to be in the center of the Eastern Cluster. However, after the first field season it was determined to be between the Central Cluster stations and the Eastern Cluster. Based on altimeter observations it is several kilometers west of the Eastern Cluster. It must dip to the north as the maximum elevation of the southern dome lies to the south. Thus, the stations in the Western and the Central Clusters should be moving west to northwest, with those farther west moving more nearly west. Similarly, the stations in the Eastern Cluster should be moving approximately northeast, with those farther east moving more nearly east.

Horizontal velocities and strain rates. The magnitude of the velocity is a function primarily of the surface slope, the ice thickness and basal sliding (Paterson, 1969). It is thought that the ice is frozen to the bed in this section of Greenland. Thus, the basal sliding should be zero. The surface of an ice sheet in equilibrium can be approximated on a large scale by a parabola (Paterson, 1980). Thus, the stations farthest from the ice crest should have the greatest slope and thus should be moving the fastest. Similarly, the strain rate is a function of the change in the slope and the change in the ice thickness. In general, the maximum strain should increase away from the ice crest. If the ice of the dome is moving uniformly, the maximum and the minimum principal strains will be extensional. However, the magnitude and sign of the minimum strain will vary greatly with the bottom topography.

Vertical motion. There are three components to the change in the elevation of a station placed on the surface of an ice sheet. One change is due to the sinking of the station due to the compaction of the snow under the station into ice. Our elevations are, however, referenced to datums at depth. Another component is due to the downslope movement of the station. This occurs because the ice on which the station is placed is in motion. This component can be calculated if the slope at the station is known. For this experiment the average slope at each cluster was used to estimate the vertical component of the downslope motion at each station. An average slope was used because the slopes at individual stations were not then available. The estimates of the vertical component of the downslope motion are listed in Table 13 (p. 45). The third and most interesting component is due to the actual change in the thickness of the ice sheet. If the measured velocity changes are sufficiently accurate and have no systematic errors, then they can be used for calculating the change in the ice sheet thickness by subtracting the downslope motion and any movement due to compaction and sinking below the datum. The change in thickness of the ice sheet is a measure of the ice balance of the ice sheet.

Data Reduction Program

An IBM version of the Doppler data reduction program GEODOP (Kouba and Boal, 1976; Archinal, 1982) was used to obtain the station positions from the Greenland data. This program uses the WGS66 ellipsoid (a=6378145 m and 1/f=298.25). Elevations found and listed in this report are above this (geocentric) ellipsoid. A tropospheric correction is used to correct the Doppler counts for atmospheric pressure, temperature and humidity. The tropospheric correction principally affects the elevations. However, the meteorologic data that were obtained at each cluster were of such poor quality as to be unusable. Meteorological data covering both the 1980 and 1981 observations were obtained from DYE-3 the U.S. Air Force radar installation within the Eastern Cluster (near station 3003). These data were used to obtain approximate atmospheric pressures, temperatures, and humidity for each of the Doppler satellite stations located on the ice sheet.

Short arc techniques were used to obtain higher accuracy than that obtainable by any other technique. Keeping two stations fixed, while solving for the other stations, minimizes the unmodeled orbital errors. The coordinates obtained for the other stations are relative to the fixed stations. Normally, six coordinates from three stations are needed for proper coordinate system definition; but in this case, since only two stations were placed at permanent locations, only five coordinates were held fixed. Thus, the coordinate system of the nonpermanent stations is not fully defined and will not necessarily be identical to the coordinate system in which the permanent stations are given.

In the adjustment in this program is a parameter for long-term drift of the oscillator. However, the short-term oscillator drift that is significant during the time of a single satellite pass can affect the results of the adjustment causing an error in the coordinates obtained. There is no way of determining whether this has occurred.

Order of Calculations

The positions of the Doppler satellite stations were determined for each year. This was done in several steps. First, the positions of the permanent stations, on the coast, were found for each year. This gives an estimate of overall accuracy and provides the "fixed" reference. Some of the stations on the ice sheet moved more than the precision obtained for the coordinates during the period that a site was occupied in one season. Therefore, it is uncertain to which date during the period of occupation the calculated coordinates refer. This effect was determined to be unimportant by analyzing the movements of a small set of stations over several short consecutive time periods.

After this, the positions of the stations on the ice were calculated. Because the Doppler observations at each cluster did not overlap with the observations at any other cluster and the locations of the coast stations were fixed, there is no loss of information in obtaining separate solutions for each cluster. Finally, velocities were determined for each station, baseline lengths for pairs of stations each year, and slopes and principal strain rates for triangles of stations.

STATIONARY RECEIVER RESULTS

Background

Two stations were located on the west coast of Greenland at stationary locations. The places of habitation in Greenland where a receiver and operator could be located are limited, thus restricting the possible locations for the fixed receivers. The station in Godthaab, operated by a project member, was located on the roof of the Roman Catholic church. The station in Søndrestrøm Fjord was operated by personnel of the Polar Ice Coring Office (PICO), the administrators of GISP and was located on a ridge within the air base jointly operated by the U.S. and Danish Air Forces. Both stations were tied to local surveying marks.

After these stations were set up in 1980, the locations of the bands on the antennas (to which the derived coordinates refer) were measured from local fixed objects. Also, a mark was painted under the antennas. Thus, in 1981, the antenna locations were recovered to within one or two centimeters. Both stations operated nearly perfectly both years. Down time due to receiver failure of the receiver or operator error was only a few days each year.

Method of Solution for Coastal Stations

The point positioning method was used to obtain the coordinates of these stations. Only those passes which were observed by both stations were used in these solutions. Because no coordinates were fixed, the positions obtained are in the coordinate system of the ephemeris used in that solution. Because the coordinates of both stations were determined in a joint solution, the coordinates obtained for the two stations are jointly correlated (see Table 6, (p. 30)). Thus, the standard deviation of the distance between the two coast stations is smaller than the standard deviations of any of the station coordinates.

Positions Obtained for Coastal Stations

The locations of the stationary stations were found for both years using both the precise and broadcast ephemeris. The broadcast ephemeris solution was obtained principally to compare the positions obtained with this ephemeris to the positions obtained with the precise ephemeris. Solutions were obtained for both years to compare the results for indications of systematic drift in the orbital errors. It was felt that the orbital errors could show up as systematic errors in the coordinates obtained for the other stations. The results are in Table 3. The locations obtained for both GOT and SFJ differ between 1980 and 1981. This difference is probably due mainly to orbital errors. Station movements here are assumed to be zero. With the precise ephemeris, the total change in location was about 0.8 meters for both stations. The change in longitude was the largest and the only component change likely to have been measurably affected by drift. With the less accurate broadcast ephemeris, the coordinates of both GOT and SFJ changed about 3 meters in latitude and longitude and about 1.5 meters in elevation. Both stations moved approximately the same amount in all three coordinates. Six orbital elements were used to define the orbits. For the precise ephemeris solutions, each orbital element was constrained by a standard deviation of two meters. For the broadcast ephemeris solutions, the orbital element of the mean motion was constrained by a standard deviation of twenty meters. The other orbital elements were constrained by standard deviations of ten meters.

TABLE 3

Results for Coastal Stations

STA.	LATITUDE	σ	LONGITUDE	σ	ELEV.	σ	SAT[1]	PASSES[2]	DAYS[3]	PASSES /DAY
	(° ′ ″)	(m)	(° ′ ″)	(m)	(m)	(m)				
PRECISE EPHEMERIS - 1980							2	311	43.9	7.1
GOT	64 10 45.919	0.18	308 16 4.483	0.22	73.06	0.19				
SFJ	67 0 9.603	0.18	309 19 29.774	0.22	255.48	0.19				
PRECISE EPHEMERIS - 1981							2	259	30.5[4]	8.5
GOT	64 10 45.920	0.18	308 16 4.439	0.23	73.62	0.20				
SFJ	67 0 9.599	0.18	309 19 29.736	0.23	255.55	0.20				
BROADCAST EPHEMERIS - 1980							5	711	40.2	17.7
GOT	64 10 45.841	0.76	308 16 4.655	0.54	76.65	0.58				
SFJ	67 0 9.506	0.77	309 19 29.946	0.56	259.10	0.57				
BROADCAST EPHEMERIS - 1981							4[5]	742	53.9	13.8
GOT	64 10 45.943	0.75	308 16 4.412	0.52	78.46	0.55				
SFJ	67 0 9.605	0.75	309 19 29.698	0.54	260.31	0.54				

[1] - SAT - The number of satellites operational (and from which data were obtained).

[2] - The number of (jointly observed) satellite passes used in the solution.

[3] - The number of days from which satellite passes were observed.

[4] - The precise ephemeris was only available for a portion of the observation period at the time that this solution was obtained.

[5] - In addition to these 4 satellites, a new satellite (#48) became operational during the last five days and contributed 22 passes to this solution.

Unit Conversion: Latitude 1.0 m = 0.″032 <=> 0.″10 = 3.10 m
Longitude 1.0 m = 0.″08 <=> 0.″10 = 1.35 m

MOVEMENT DURING EACH OCCUPATION PERIOD

This part of the research was designed to determine how the movement of a satellite receiver during the time of one occupation (because of the continuous movement of the ice) would affect the results (positions, variances, etc.). Observations from five stations were used for this analysis. The broadcast ephemeris was used for obtaining the positions. The coordinates of the two coastal stations were fixed (their variances were set to 1 mm). Three stations in the Western Cluster were also used (1001, 1002 and 1005). These stations were occupied continuously for the entire occupation of the Western Cluster. They include the fastest moving station (1005) which moved 46.5 meters per year. The other two stations were directly east of 1005 and were moving at 35.8 meters per year (station 1001) and at 30.0 meters per year (station 1002).

During a fifteen-day time period station 1005 moved nearly two meters, while 1001 and 1002 moved about 1.5 and 1.25 meters respectively. As the expected error is less than 0.20 meters with the short arc method, these movements should affect the coordinates obtained for those stations. The fifteen-day period was divided into five three-day periods and also into three five-day periods. A solution was obtained for each of these time periods as well as for the whole fifteen-day period. Thus, nine solutions were obtained. These solutions are not independent, primarily because the same data were used in more than one solution. The amount of correlation, however, is unknown. But solutions which include the same days must be very highly correlated, while those which do not include common days, are less so. For instance, the fifteen-day solution includes all the data used in each of the other solutions, thus coordinates obtained in the solution must be highly correlated with the coordinates obtained in every other solution.

The results are shown in Table 4. The coordinates for the five-day and the three-day periods are not explicitly given. Instead, the differences between these coordinates and the coordinates obtained in the fifteen-day solution are given. It is possible to see a trend in the coordinates. A linear fit was made for latitude versus time, longitude versus time, and elevation versus time in the three-day and the five-day solutions. From this, coordinates for the median of the fifteen-day period were obtained. Table 5 lists these coordinates, the fifteen-day period coordinates and their differences. The differences in latitude and elevation seem insignificantly small while the differences in longitude are consistently slightly low. There is no readily apparent explanation for this. The overall average of the differences is less than 0.01 meters. Thus, it appears that the median of the period of observation is acceptable for use as the date to which the final results can be said to refer.

TABLE 4

Movement Analysis Results

STA.	OBS. PERIOD	LATITUDE	LONGITUDE	ELEVATION	SAT. PASSES
		(° ′ ″)	(° ′ ″)	(m)	
1001	1-15	65 23 15.796	312 19 34.703	2025.98	315
1002	1-15	65 23 12.569	312 45 41.415	2165.97	305
1005	1-15	65 23 25.728	311 53 21.791	1863.94	340
		DIF. IN LATITUDE	DIF. IN LONGITUDE	DIF. IN ELEVATION	
		(m)	(m)	(m)	
1001	1-5	0.18	-0.35	-0.77	123
	6-10	0.19	-0.22	0.14	87
	11-15	-0.29	0.19	0.26	105
1002	1-5	0.45	-0.08	-0.99	97
	6-10	-0.05	-0.51	0.11	103
	11-16	-0.43	0.33	0.55	105
1005	1-5	0.20	-0.43	-1.11	132
	6-10	-0.02	-0.31	0.34	120
	11-16	-0.31	0.61	0.11	88
1001	1-3	0.15	-1.20	-0.18	73
	4-6	0.56	-0.24	-0.52	78
	7-9	-0.03	-0.34	0.47	46
	10-12	-0.09	0.11	-0.18	54
	13-15	-0.57	0.20	0.78	61
1002	1-3	0.54	-1.38	-0.39	58
	4-6	0.19	0.12	-0.50	66
	7-9	0.01	-0.31	0.09	50
	10-12	-0.38	-0.24	0.69	65
	13-15	-0.30	0.37	0.07	62
1005	1-3	0.20	-1.44	-1.04	80
	4-6	0.49	0.11	0.21	72
	7-9	0.08	-0.39	0.40	69
	10-12	-0.40	0.49	0.00	60
	13-15	-0.08	0.18	0.51	52

TABLE 5

Straight Line Fit to Movement Analysis Results

STA.	OBSERVATION LENGTH		COORDINATES	DIFFERENCE
				(m)
1001	15	LAT	65 23 15.796	
	5	(° ′ ″)	15.792	0.12
	3		15.962	0.002
	15	LONG	312 19 34.703	
	5	(° ′ ″)	34.716	-0.18
	3		34.725	-0.29
	15	ELEV	2025.98	
	5	(m)	2026.19	0.79
	3		2025.91	0.07
1002	15	LAT	65 23 12.569	
	5	(° ′ ″)	12.565	0.13
	3		12.569	0.01
	15	LONG	312 45 41.415	
	5	(° ′ ″)	41.430	-0.20
	3		41.437	-0.29
	15	ELEV	2165.97	
	5	(m)	2166.26	0.29
	3		2165.98	-0.01
1005	15	LAT	65 23 25.728	
	5	(° ′ ″)	25.727	0.06
	3		25.727	0.06
	15	LONG	312 53 21.791	
	5	(° ′ ″)	21.810	-0.25
	3		21.807	-0.21
	15	ELEV	1863.94	
	5	(m)	1864.20	-0.26
	3		1863.92	0.02

CLUSTER RESULTS

Selection of Fixed (Coastal) Coordinates and Their Effect

The results from the precise ephemeris solution for the coastal stations with the 1980 data (see Table 3) were used for the coordinates of SFJ and GOT. The output weight matrix from that solution was used to obtain a variance-covariance matrix for these two stations. To obtain the variances for the five fixed coordinates, (the X, Y and Z coordinates of SFJ and the X and Y coordinates of GOT) their variances were divided by 10^4 (thus dividing the standard deviations by 100). The variance of the coordinate not fixed (the Z coordinate of GOT) was not changed. The covariances were changed so that the correlations would not be changed. The coordinates used, their standard deviations and values of the correlations are given in Table 6.

Because one coordinate (the Z coordinate of GOT) was not fixed, different values were obtained for that coordinate for each solution (each cluster, each year). The average change in the Z coordinate from the input value (1980 precise ephemeris value) was 0.60 m/yr. Because the station did not actually change position by more than a few centimeters at most, this change must be caused by a change in the coordinate system. The Z coordinate of GOT changed in the same direction both years in all solutions (each cluster, each year). The differences in the Z coordinate results between 1980 and 1981 averaged only 0.28 meters. This is statistically insignificant when compared with the standard deviation of 0.21 m given to this coordinate in the input of each solution.

TABLE 6

Fixed Coordinates, Standard Deviations and Correlations

COORDINATES

STA	X	σ	Y	σ	Z	σ
	(m)	(m)	(m)	(m)	(m)	(m)
GOT	1725248.4	0.0025	-2187061.0	0.0025	5718527.4	0.2063
SFJ	1583701.1	0.0025	-1933187.0	0.0025	5848771.5	0.0020

CORRELATIONS

GOT			SFJ				
X	Y	Z	X	Y	Z		
1.00	0.09	0.14	0.81	0.03	0.13	X	GOT
	1.00	-0.18	0.05	0.84	-0.17	Y	
		1.00	0.09	-0.17	0.85	Z	
			1.00	0.11	0.14	X	SFJ
				1.00	-0.19	Y	
					1.00	Z	

Input of Nonfixed Coordinates

The initial assumed values for the coordinates of the stations on the ice sheet were the point positions calculated by the satellite receiver while in position. These coordinates were given standard deviations of 25 meters with no correlation. Also given as input for the second year were the vertical offsets determined from the datums. Thus, the coordinates listed as results have had this effect already removed, i.e., they are referred to the 1980 station locations.

Results

General results. Tables 10, 11 and 12 list the latitudes, longitudes and elevations and their formal standard deviations which were obtained using both 1980 and 1981 data. The differences in the coordinates between the two years (1981 minus 1980) are also given. Assuming that the median date of the occupation of a cluster is the date to which the coordinates refer, and that there are no other significant, nonrandom errors, then the velocities can be obtained from the observed differences of the coordinates. The fractions of a year between successive occupations of each cluster were 0.979 for Western Cluster, 1.005 for Central Cluster and 0.998 for the Eastern Cluster. The horizontal velocities and the azimuths of movement for each station along with their standard deviations are given in Table 7 and are shown in Figures 5, 6 and 7. Slopes were calculated for each triangle of stations from the 1980 elevations. They are listed in Table 8.

Deformation may be measured by using either measurements of coordinates, distances or angles. The distance method was used in this report because distances are invariant with respect to rotations or translations of the coordinate system. Consequently, the rotational component cannot be calculated. The strain tensor is a 2x2 matrix, expressed in terms of orthogonal (x,y) coordinates. The components of the strain tensor are (Livieratos, 1980):

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix}$$

where, $\varepsilon_{xx} = \frac{\partial u}{\partial x}$, $\varepsilon_{yy} = \frac{\partial v}{\partial y}$ and, $\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$

where u is the displacement in the x direction, and v is the displacement in the y direction.

TABLE 7

Station Velocities

STA.	HORIZONTAL VELOCITY (m/yr)	σ (m/yr)	AZIMUTH (° ′)		σ (° ′)		VERTICAL VELOCITY (m/yr)	σ (m/yr)
WESTERN CLUSTER								
1001	35.76	0.12	288	6	0	8	-0.02	0.12
1002	30.04	0.14	289	10	0	11	-1.20	0.13
1003	36.07	0.15	290	17	0	10	0.37	0.14
1004	42.08	0.14	287	17	0	18	-0.21	0.15
1005	46.50	0.12	288	33	0	6	-0.10	0.11
1006	44.05	0.24	285	0	0	14	1.17	0.21
1007	33.99	0.20	287	3	0	13	0.21	0.16
CENTRAL CLUSTER								
2001	9.32	0.12	297	40	0	33	-0.55	0.11
2002	7.55	0.11	309	20	0	47	-0.15	0.13
2003	8.58	0.13	298	14	0	40	0.50	0.13
2004	11.38	0.13	292	48	0	27	0.04	0.12
2005	11.56	0.13	294	53	0	25	0.55	0.11
2006	10.95	0.14	301	44	0	35	-0.22	0.13
2007	8.21	0.14	306	46	0	51	0.08	0.13
EASTERN CLUSTER								
3001	27.43	0.16	68	18	0	15	-0.53	0.16
3002	23.20	0.16	62	37	0	18	-1.84	0.16
3003	13.44	0.14	63	49	0	27	-0.42	0.14
3004	13.65	0.12	55	35	0	26	0.86	0.14
3005	7.57	0.11	52	29	0	44	-0.05	0.14
3006	6.81	0.15	49	21	1	11	-0.11	0.16
3007	2.42	0.11	339	10	3	33	-0.82	0.15
3008	3.06	0.10	348	6	2	54	-0.19	0.14

TABLE 8

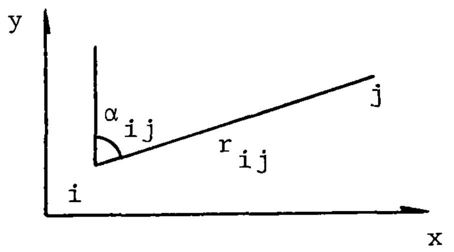
Strain Rates and Slopes for Small Triangles of Stations

STATIONS	MAXIMUM STRAIN RATE	MINIMUM STRAIN RATE	MAXIMUM STRAIN AZIMUTH	MAXIMUM SLOPE[2]	SLOPE AZIMUTH
	(ppm/yr[1])	(ppm/yr[1])	(°)	(%)	(°)
WESTERN CLUSTER					
1001-1002-1003	278.4	-111.5	264.1	-0.73	286.4
1001-1003-1004	349.1	-94.3	259.0	-0.74	285.7
1001-1004-1005	498.8	12.5	275.8	-0.83	287.3
1001-1005-1006	547.7	-149.2	287.0	-0.82	284.3
1001-1006-1007	528.2	-61.2	284.3	-0.67	298.2
1001-1007-1002	290.5	-47.8	282.8	-0.80	298.4
CENTRAL CLUSTER					
2001-2002-2003	125.4	19.2	265.0	-0.36	298.7
2001-2003-2004	126.1	9.5	276.0	-0.39	294.9
2001-2004-2005	114.5	20.1	276.6	-0.39	294.8
2001-2005-2006	113.7	60.9	287.6	-0.40	297.9
2001-2006-2007	138.8	56.8	283.1	-0.37	301.6
2001-2007-2002	128.3	12.7	263.5	-0.37	301.5
EASTERN CLUSTER					
3001-3002-3003	756.2	-124.1	70.9	-0.61	53.9
3002-3003-3004	526.9	-119.7	78.9	-0.40	66.6
3003-3004-3005	357.7	-109.7	79.5	-0.36	62.8
3004-3005-3006	394.0	-26.3	71.5	-0.48	54.5
3005-3006-3007	318.8	-7.8	75.7	-0.34	36.5
3006-3007-3008	268.7	-44.5	84.5	-0.25	41.3

1 - Strain is in parts per million (ppm) per year. Positive strain indicates expansion.

2 - Slope is calculated from 1980 coordinates. The negative sign indicates that the surface is sloping downward in the direction of the azimuth of the (maximum) slope.

The diagram and equation below give the relationship between the measured, known and unknown elements.



$$\varepsilon_{r_{ij}} = \sin^2 \alpha_{ij}\ \varepsilon_{xx} + \sin 2\alpha_{ij}\ \varepsilon_{xy} + \cos^2 \alpha_{ij}\ \varepsilon_{yy}$$

where, $\varepsilon_{r_{ij}} = \dfrac{\partial r_{ij}}{r_{ij}}$ and α_{ij} is the azimuth of r_{ij}.

Because ε_{xx} , ε_{xy} and ε_{yy} are the unknowns, three equations are needed to obtain a solution.
Thus, the distances between a triangle of three stations is the minimum number of measurements needed.

From the strains in the x and y directions and the shear across x and y, the maximum strain (ε_{max}), the minimum strain (ε_{min}) and the azimuth of the (maximum) strain (ϕ) can be calculated as follows (Livieratos, 1980):

$$\varepsilon_{max} = \tfrac{1}{2}\left[\varepsilon_{xx} + \varepsilon_{yy} + \sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + (2\varepsilon_{xy})^2}\right]$$

$$\varepsilon_{min} = \tfrac{1}{2}\left[\varepsilon_{xx} + \varepsilon_{yy} - \sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + (2\varepsilon_{xy})^2}\right]$$

$$\phi = \tfrac{1}{2}\tan^{-1}\left\{\frac{2\varepsilon_{xy}}{\varepsilon_{xx} - \varepsilon_{yy}}\right\}$$

The strains can be differentiated with respect to time to give strain rates. The distances used in this report are baseline lengths on the ellipsoid. They were calculated using the Gauss mid-latitude formulas (Rapp, 1979) and are listed in Table 9. The differences in the baseline lengths between years were normalized to one year. Thus, average yearly strain rates were calculated, rather than the strain between observations. Because strain is a ratio of deformation, the strain on the projection of the stations to the ellipsoid is the same as the strain calculated on an ellipsoid at the elevation of the stations. Because the baselines are geodesics on the ellipsoid, the calculation of the strain using them is virtually the same as calculating the strain on a plane tangent to the ellipsoid at the geometric center of the triangle of stations. To the accuracy needed for this report, these strains are the same as strains calculated on a plane passed through the three station positions. Strain rates for the small triangles are listed in Table 8 and plotted in Figures 5, 6 and 7. As can be seen from the plots, the velocities tend to be more northerly than the directions of maximum strain. Both the velocities and the strain rates increase on both sides from the ice crest. However, the velocities and the maximum strain rates east of the ice crest increase more rapidly than those to the west.

Correlations in the results. The correlation coefficients between the coastal station coordinates and the coordinates of the cluster stations were between -0.05 and +0.20. Of these the highest correlations were between the latitude and elevation of GOT and the latitudes and elevations of the cluster stations. The correlations between the latitude and elevation of GOT was greater than +0.99. Among the coordinates of the cluster stations, the highest correlations were between the same coordinates from different stations. Elevation-elevation correlations generally ranged between +0.60 and 0.80, while latitude-latitude and longitude-longitude correlations were generally between +0.30 and +0.60.

In all cases, the correlations between stations operating simultaneously were somewhat higher than between those not observing at the same time. Other correlations were between -0.10 and +0.10, with the larger correlations between latitudes and elevations, and between stations operating simultaneously. The high correlations between the elevations of stations within a cluster may have had an effect on the magnitude of the elevation differences described at page 44.

TABLE 9

Baseline Lengths

STATIONS	1980 DISTANCE	1980 σ	1981 DISTANCE	1981 σ	DIFFERENCES 1981-1980
	(m)	(m)	(m)	(m)	(m)
WESTERN CLUSTER					
1001-1002	20227.53	0.10	20232.96	0.08	5.43
1001-1003	19504.75	0.07	19503.91	0.08	-0.84
1001-1004	18645.40	0.09	18647.09	0.08	1.69
1001-1005	20304.34	0.09	20314.18	0.08	9.84
1001-1006	20528.98	0.16	20533.53	0.09	4.55
1001-1007	19579.93	0.12	19579.58	0.08	-0.35
1002-1003	19624.60	0.08	19643.27	0.09	0.67
1003-1004	19208.65	0.12	19214.80	0.13	6.15
1004-1005	19918.30	0.09	19922.40	0.08	4.10
1005-1006	20077.55	0.15	20075.26	0.09	-2.29
1006-1007	20018.20	0.22	20028.03	0.14	9.83
1007-1002	20001.51	0.12	20003.77	0.08	2.26
CENTRAL CLUSTER					
2001-2002	19332.95	0.08	19335.18	0.10	2.23
2001-2003	20001.72	0.07	20002.14	0.05	0.42
2001-2004	21343.52	0.09	21344.86	0.07	1.34
2001-2005	20094.04	0.09	20096.33	0.08	2.29
2001-2006	21727.65	0.07	21729.38	0.07	1.73
2001-2007	20208.74	0.09	20210.24	0.08	1.50
2002-2003	20924.32	0.08	20926.01	0.08	1.69
2003-2004	21015.35	0.11	21017.95	0.09	2.60
2004-2005	19036.67	0.06	19037.22	0.05	0.55
2005-2006	20401.32	0.08	20402.70	0.08	1.38
2006-2007	20507.50	0.11	20510.36	0.11	2.86
2007-2002	20821.10	0.06	20821.43	0.08	0.33

TABLE 9 - CONTINUED

STATIONS	1980 DISTANCE	1980 σ	1981 DISTANCE	1981 σ	DIFFERENCES 1981-1980
	(m)	(m)	(m)	(m)	(m)
EASTERN CLUSTER					
3001-3002	17027.59	0.05	17027.05	0.09	-0.54
3001-3003	18585.26	0.08	18599.28	0.12	14.02
3002-3003	20541.93	0.08	20546.68	0.12	4.75
3002-3004	20125.95	0.08	20135.59	0.12	9.64
3003-3004	19633.99	0.04	19632.21	0.06	-1.78
3003-3005	19142.09	0.07	19148.02	0.09	5.93
3004-3005	19461.86	0.07	19464.59	0.08	2.73
3004-3006	19160.89	0.10	19167.75	0.12	6.86
3005-3006	21439.90	0.06	21440.04	0.08	0.14
3005-3007	23985.35	0.10	23992.46	0.09	7.11
3006-3007	23963.63	0.12	23968.21	0.12	4.58
3006-3008	24001.95	0.11	24007.84	0.13	5.89
3007-3008	18331.71	0.07	18330.98	0.06	-0.73

Note: The fractions of a year between observations are;

WESTERN CLUSTER: 0.979 yr.
CENTRAL CLUSTER: 1.005 yr.
EASTERN CLUSTER: 0.998 yr.

Description of Western Cluster results. The velocities of the Western Cluster stations are nearly parallel and lie between azimuths of 285 and 290 degrees, but the direction of maximum strain rates diverge in the direction of flow. The average azimuth of the maximum strain rates is 275 degrees. Thus, the ice is flowing about 12 degrees north of the average direction of maximum extension. The minimum strains are almost all negative. Thus, the ice in this cluster is expanding more or less along the flow of the ice and contracting perpendicular to the flow. Several stations (1002 and 1006) show anomalously large elevation differences.

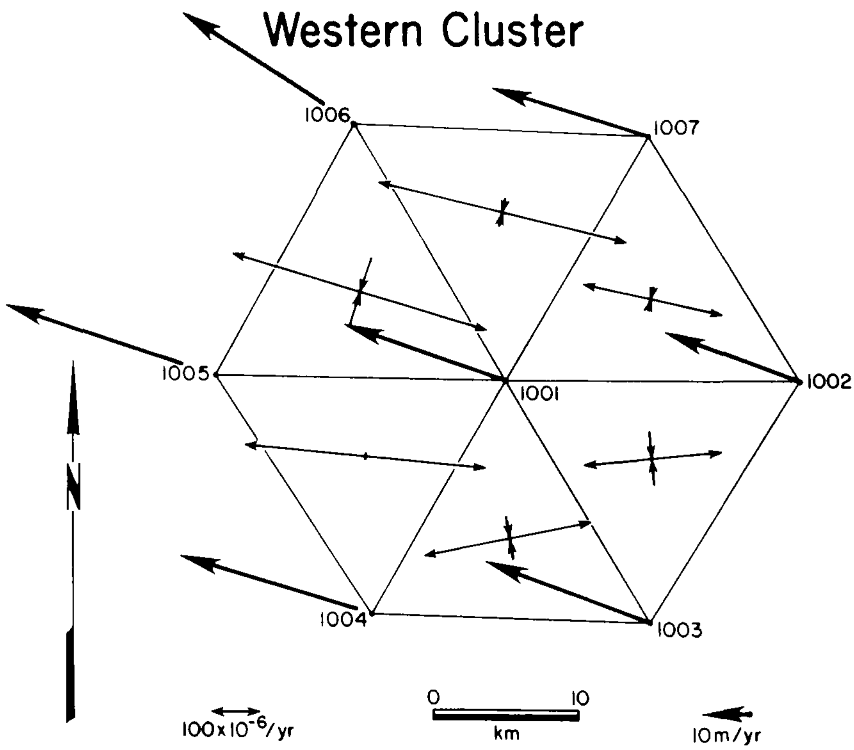


Illustration 5: Western Cluster Velocities and Strain Rates

TABLE 10

Western Cluster Results - Short Arc Method, All Stations

STA.	YR	LATITUDE	σ	LONGITUDE	σ	ELEVATION	σ	PASSES
		(° ′ ″)	(m)	(° ′ ″)	(m)	(m)	(m)	
1001	80	65 23 15.484	0.06	312 19 37.248	0.09	2022.08	0.09	325
	81	15.835	0.05	34.670	0.08	2022.06	0.07	368
	dif.	10.88 m		-33.28 m		-0.02		
1002	80	65 23 12.297	0.07	312 45 43.515	0.11	2163.52	0.10	264
	81	12.609	0.05	41.364	0.08	2162.35	0.07	346
	dif.	9.66 m		-27.78 m		-1.17		
1003	80	65 14 13.406	0.06	312 32 23.829	0.10	2126.08	0.09	197
	81	13.795	0.07	21.279	0.11	2126.44	0.10	174
	dif.	12.24 m		-33.12 m		0.36		
1004	80	65 14 32.185	0.07	312 37 45.415	0.09	1988.80	0.08	183
	81	32.581	0.06	42.386	0.11	1988.59	0.09	183
	dif.	12.24 m		-39.34 m		-0.21		
1005	80	65 23 25.301	0.05	311 53 25.082	0.09	1860.16	0.08	340
	81	25.769	0.05	21.739	0.08	1860.06	0.07	361
	dif.	14.48 m		-43.16 m		-0.10		
1006	80	65 32 48.673	0.14	312 6 16.551	0.20	1904.54	0.10	53
	81	49.033	0.08	13.306	0.12	1905.69	0.10	134
	dif.	11.16 m		-41.66 m		1.15		
1007	80	65 32 23.638	0.10	312 32 14.732	0.16	2025.62	0.13	102
	81	23.953	0.06	12.255	0.11	2025.83	0.09	207
	dif.	9.76 m		-31.81 m		0.21		
GOT	80	64 10 45.918	0.03	308 16 4.461	0.00	73.80	0.07	243
	81	45.922	0.02	4.461	0.00	74.08	0.05	440
	dif.	0.13 m		0.00 m		0.28		
SFJ	80	67 0 9.610	0.00	309 19 29.794	0.00	255.36	0.00	282
	81	9.610	0.00	29.794	0.00	255.36	0.00	437
	dif.	0.00 m		0.00 m		0.00		

Unit Conversion: Latitude 1.0 m 0.″032 <=> 0.″10 = 3.10 m
Longitude 1.0 m 0.″08 <=> 0.″10 = 1.35 m

Description of Central Cluster results. The Central Cluster velocities have a much smaller magnitude than those of the Western Cluster stations. They are also not as close to parallel as the Western Cluster velocities. Their average azimuth is 300 degrees. The strain rates at the Central Cluster are also smaller than those of the Western Cluster. The pattern of divergence of the maximum strain rates at this cluster is similar to that of the Western Cluster and their average direction is the same. Thus, the ice is moving nearly 25 degrees from the direction of maximum strain. However, unlike the Western Cluster, the minimum strain rates are all positive. In other words, the ice in this cluster is expanding in all horizontal directions. While none of the elevation differences are as large as in the Western Cluster, several are well outside the expected range.

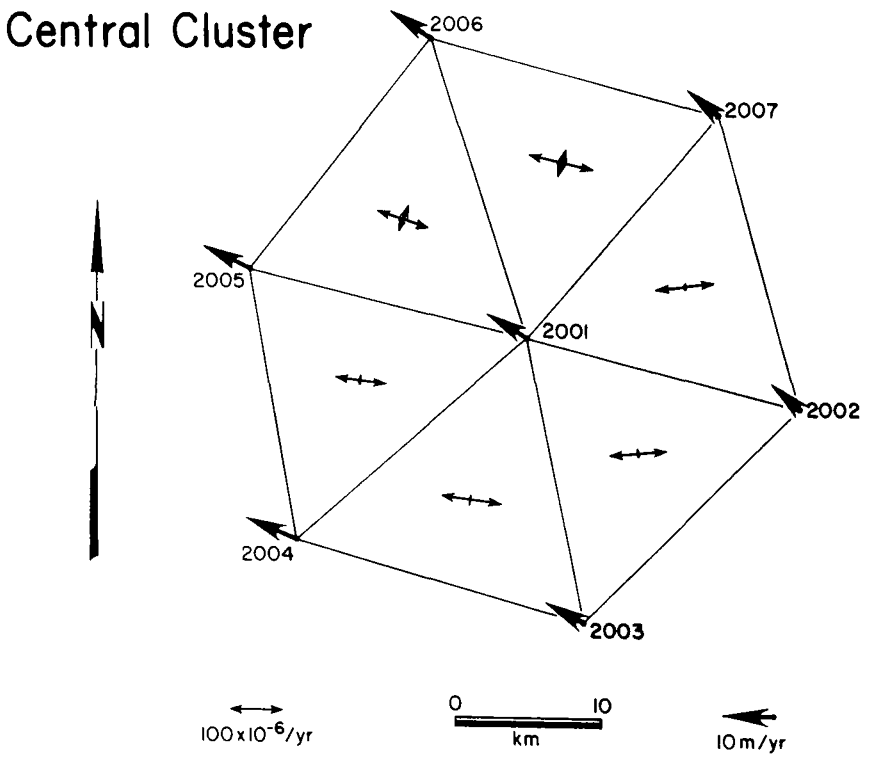


Illustration 6: Central Cluster Velocities and Strain Rates

TABLE 11

Central Cluster Results - Short Arc Method, All Stations

STA.	YR	LATITUDE	σ	LONGITUDE	σ	ELEVATION	σ	PASSES
		(° ′ ″)	(m)	(° ′ ″)	(m)	(m)	(m)	
2001	80	65 6 29.389	0.06	314 18 48.287	0.10	2518.72	0.09	286
	81	29.529	0.02	47.651	0.08	2518.17	0.07	314
	dif.	4.35 m		-8.30 m		-0.55		
2002	80	65 4 2.070	0.05	314 42 46.644	0.09	2586.13	0.09	322
	81	2.225	0.06	46.195	0.10	2585.98	0.09	161
	dif.	4.81 m		-5.87 m		-0.15		
2003	80	64 55 58.236	0.07	314 24 12.152	0.11	2566.12	0.10	139
	81	58.368	0.05	11.574	0.09	2566.62	0.08	199
	dif.	4.08 m		-7.60 m		0.50		
2004	80	64 58 53.295	0.06	313 58 25.216	0.11	2485.04	0.09	171
	81	53.438	0.05	24.438	0.09	2485.08	0.09	223
	dif.	4.43 m		-10.54 m		0.04		
2005	80	65 8 56.704	0.05	313 53 47.725	0.11	2441.96	0.08	261
	81	56.863	0.05	46.916	0.08	2442.51	0.07	384
	dif.	4.89 m		-10.54 m		0.55		
2006	80	65 17 34.619	0.07	314 9 57.747	0.11	2456.72	0.10	116
	81	34.807	0.06	57.025	0.11	2456.50	0.09	156
	dif.	5.79 m		-9.36 m		-0.22		
2007	80	65 14 48.551	0.07	314 35 28.130	0.11	2529.63	0.10	128
	81	48.711	0.07	27.621	0.11	2529.71	0.09	150
	dif.	4.94 m		-6.61 m		0.08		
GOT	80	64 10 45.916	0.03	308 16 4.461	0.00	73.70	0.05	337
	81	45.921	0.03	4.461	0.00	73.99	0.05	333
	dif.	0.14 m		0.00 m		0.29		
SFJ	80	67 0 9.610	0.00	309 19 29.794	0.00	255.36	0.00	308
	81	9.610	0.00	29.794	0.00	255.36	0.00	434
	dif.	0.00 m		0.00 m		0.00		

Unit Conversion: Latitude 1.0 m = 0".032 <=> 0".10 = 3.10 m
Longitude 1.0 m = 0".08 <=> 0".10 = 1.35 m

Description of Eastern Cluster results. The two stations in the Eastern Cluster that are nearest the ice crest (3007 and 3008) are moving in a distinctly different direction than the other stations in this cluster. They are moving slightly west of north at about 345 degrees. The azimuths of the movements of the other stations are between 50 and 70 degrees with those farther from the ice divide moving more nearly east. Thus, the two westernmost stations in this cluster are on the east side of the ice crest (the line of the highest elevations), but west of the ice divide (the line of ice flow divergence). There is an obvious rotation in the velocity vectors over the cluster. The strain rates at this cluster do not show this rotation. They also do not show the same type of diverging pattern as the strain rates at the other clusters, but are more nearly parallel. Their average azimuth is 77 degrees. As in both the Western Cluster and the Central Cluster, the ice movement is north of the direction of maximum extension. The minimum strains are all compressive. Again, as in the other clusters, several of the stations (3002 and 3004) have anomalous elevation differences.

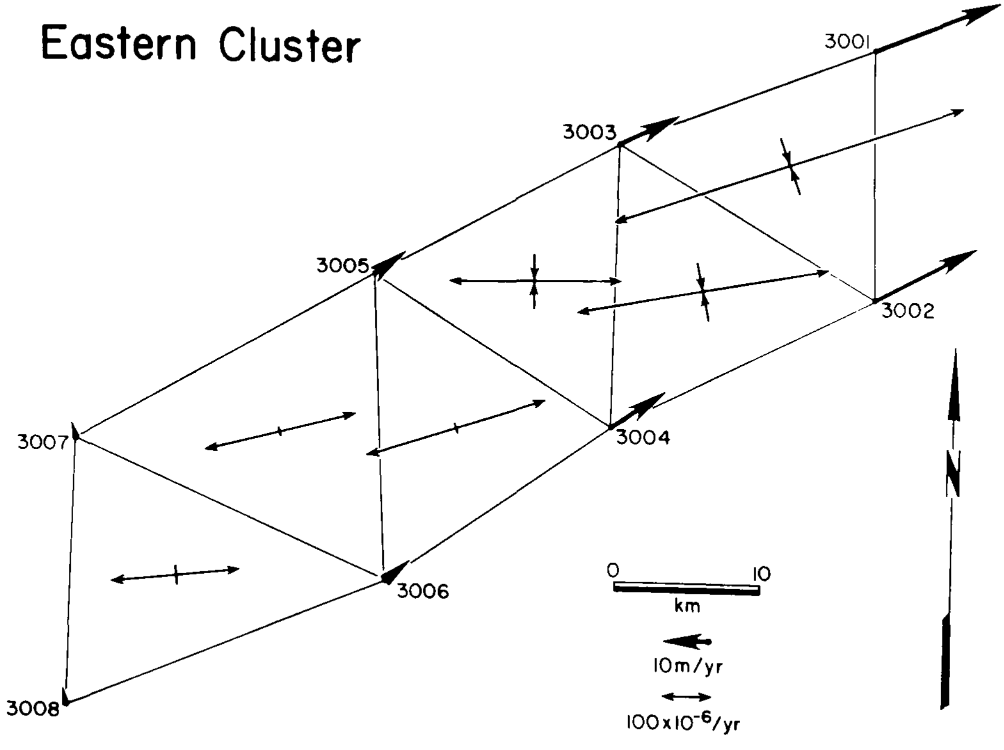


Illustration 7: Eastern Cluster Velocities and Strain Rates

TABLE 12

Eastern Cluster Results - Short Arc Method, All Stations

STA.	YR	LATITUDE	σ	LONGITUDE	σ	ELEVATION	σ	PASSES
		(° ′ ″)	(m)	(° ′ ″)	(m)	(m)	(m)	
3001	80	65 15 31.992	0.06	316 31 58.454	0.09	2414.35	0.09	284
	81	32.319	0.09	32 0.414	0.14	2413.82	0.13	89
	dif.	10.12 m		25.44 m		-0.53		
3002	80	65 6 22.205	0.06	316 32 4.242	0.09	2474.59	0.09	315
	81	22.549	0.09	5.817	0.14	2472.75	0.13	73
	dif.	10.65 m		20.56 m		-1.84		
3003	80	65 12 5.512	0.05	316 9 35.228	0.08	2522.35	0.08	342
	81	5.703	0.08	36.154	0.12	2521.93	0.12	140
	dif.	5.92 m		12.04 m		-0.42		
3004	80	65 1 31.669	0.05	316 9 7.069	0.08	2555.62	0.08	443
	81	31.918	0.07	7.927	0.11	2556.48	0.11	233
	dif.	7.70 m		11.24 m		0.86		
3005	80	65 7 7.281	0.05	315 48 8.072	0.07	2590.89	0.08	446
	81	7.430	0.06	8.531	0.10	2590.84	0.11	273
	dif.	4.60 m		5.99 m		-0.05		
3006	80	64 55 35.544	0.07	315 49 12.478	0.11	2648.16	0.10	130
	81	35.687	0.09	12.871	0.13	2648.05	0.13	87
	dif.	4.43 m		5.16 m		-0.11		
3007	80	65 0 50.911	0.07	315 21 24.421	0.11	2665.58	0.10	145
	81	50.984	0.07	24.356	0.11	2664.76	0.11	137
	dif.	2.26 m		-0.86 m		-0.82		
3008	80	64 50 59.189	0.06	315 20 48.392	0.10	2700.68	0.09	151
	81	59.285	0.08	48.345	0.12	2701.05	0.11	116
	dif.	2.99 m		-0.63 m		-0.19		
GOT	80	64 10 45.915	0.02	308 16 4.461	0.00	73.62	0.05	432
	81	45.918	0.04	4.461	0.00	73.82	0.08	148
	dif.	0.10 m		0.00 m		0.20		
SFJ	80	67 0 9.610	0.00	309 19 29.794	0.00	255.36	0.00	426
	81	9.610	0.00	29.794	0.00	255.36	0.00	201
	dif.	0.00 m		0.00 m		0.00		

Unit Conversion: Latitude 1.0 m = 0".032 <=> 0".10 = 3.10 m
Longitude 1.0 m 0".08 <=> 0".10 = 1.35 m

Discussion of elevation differences. The vertical velocities are shown in Table 7. The formal standard deviations of these velocities are around 0.14 meters assuming no correlations between years. A few of the values shown are well outside the expected range of vertical velocities. Because of this, two more sets of solutions were calculated for the station coordinates. In the first of these solutions, the data from the coastal stations were not used, because it was felt that the relatively long distances between the fixed stations and the cluster stations might not insure that the elevations would be determined accurately. That is, because the two lines between the coast stations and any station on the ice sheet were nearly horizontal, the vertical coordinate may be less well constrained than the horizontal coordinates. In these solutions station positions were determined with respect to a station within the same cluster. This set of solutions, however, still shows the same kind of anomalously large vertical differences, as in the previous solution, usually for the same stations. This indicates that the amomalously large vertical differences were not, in fact, a result of the weaker constraint on the elevations.

In the second of the two additional solutions the point positioning method was used for each station with the precise ephemeris. This eliminated the effect of any station constraint on the solution for any other station position. In this third solution, the largest differences in the elevations between the two years are even larger than in the previous two solution sets. However, they are, in general, at the same stations as in the previous two solutions. Thus, some other type of explanation must be found for the anomalously large vertical velocities. The results of these two sets of solutions are described further in the next section. Table 13 shows the elevation differences obtained for all three sets of solutions along with some other factors that may have caused the anomalously large vertical velocities.

Because anomalously large vertical velocities are present in all three types of solutions, other factors that might have affected the elevations were considered. These are antenna offsets, receiver delays, oscillator drift, large scale changes in the ice sheet thickness, downslope motion and possible rotation of the coordinate system. The sinking of the ice due to further accumulation of snow and

TABLE 13

Elevation Differences - All Solutions

STA.	RECEIVER #[1] 80	RECEIVER #[1] 81	ANTENNA OFFSET (81-80)[2]	VERTICAL DOWNSLOPE MOTION[3]	SHORT ARC, ALL STATIONS EL. DIF.	σ	SHORT ARC, CLUSTER STATIONS EL. DIF.	σ	POINT POSITION, PRECISE EPHEMERIS EL. DIF.	σ	SIMUL. OCCUP.[4]	
			(m)	(m/yr)	(m)	(m)	(m)	(m)	(m)	(m)		
WESTERN CLUSTER												
1001	28	33	1.06	-0.26	-0.02	0.11	-0.04	0.00	-0.42	0.43	*	*
1002	107	42	0.89	-0.23	-1.17	0.12	-1.45	0.10	-2.29	0.45	*	*
1003	42	107	1.12	-0.26	-0.36	0.14	-0.37	0.11	-0.35	0.59	*	
1004	109	108	0.41	-0.31	-0.21	0.12	-0.46	0.11	-0.98	0.63	*	
1005	25	113	0.89	-0.34	-0.10	0.11	-0.38	0.09	-0.29	0.41	*	*
1006	109	108	0.06	-0.32	1.15	0.21	1.29	0.20	0.15	1.07		*
1007	42	107	0.91	-0.25	0.21	0.16	0.09	0.14	-1.91	0.73		*
GOT	102	35	0.00		0.28	0.09	-	-	-	-		
SFJ	43	43	0.00	-	0.00	0.00		-	-	-		
CENTRAL CLUSTER												
2001	107	113	0.35	-0.04	-0.55	0.11	-0.50	0.00	-1.00	0.40	*	*
2002	28	108	0.37	-0.03	-0.15	0.13	0.50	0.09	-1.46	0.44	*	*
2003	42	33	0.62	-0.03	0.50	0.13	1.33	0.10	-0.71	0.59	*	
2004	25	42	0.85	-0.04	0.04	0.12	0.48	0.09	0.34	0.51	*	
2005	109	107	0.60	-0.04	0.55	0.11	0.89	0.09	0.82	0.45	*	*
2006	25	42	0.44	-0.04	-0.22	0.14	0.51	0.11	0.43	0.81		*
2007	42	33	0.33	-0.03	0.08	0.14	0.89	0.11	-0.13	0.61		*
GOT	102	35	0.00		0.29	0.07	-	-	-	-		
SFJ	43	43	0.00		0.00	0.00						
EASTERN CLUSTER												
3001	109	108/33	0.51	-0.11	-0.53	0.16	-0.47	0.11	0.74	0.54	*	
3002	107	33/108	0.60	-0.09	-1.84	0.16	-1.82	0.11	-3.53	0.58	*	
3003	28	42	0.41	-0.05	-0.42	0.14	-0.62	0.09	-0.14	0.44	*	
3004	42	107	0.36	-0.06	0.86	0.14	0.68	0.08	3.75	0.43	*	*
3005	25	113	0.21	-0.03	-0.05	0.14	0.04	0.00	0.27	0.32	*	*
3006	109	108	0.27	-0.03	-0.11	0.16	0.02	0.11	0.25	0.58		*
3007	107	42	0.51	-0.01	-0.82	0.15	-0.78	0.11	-2.23	0.55		*
3008	28	33	0.27	-0.01	-0.19	0.14	0.36	0.11	0.00	0.51		*
GOT	102	35	0.00	0.00	0.20	0.09	-	-	-	-		
SFJ	43	43	0.00	0.00	0.00	0.00	-	-	-	-		

Notes:

1 Receiver # - The instrument number of the receiver at the site each year.

2 Antenna Offset - Difference in height of 1981 antenna position with respect to the 1980 position.

3 Vertical Downslope Motion - Horizontal velocity times the average slope for each cluster.

Average slopes:
- Western Cluster -0.760 %
- Central Cluster -0.375 %
- Eastern Cluster -0.410 %

4 - Simultaneous Occupation - Station in the same cluster with asterisks in the same column were occupied simultaneously. GOT and SFJ were occupied simultaneously with all stations.

the compaction of the snow beneath the station into ice has, in theory, been removed from these results with the antenna offset. However, there may be blunders in the offsets.

One of the possible factors is the electrical and electronic components of the receivers and antennas. It was assumed that this parameter was the same for each receiver, antenna and the electrical cables connecting them. This parameter is called the receiver delay. Another factor is the drift of the oscillator. It appears that one of these two factors may be a problem in one or more of these receivers. The instrument numbers of each receiver were recorded in the field. These numbers are listed for each year in Table 13. The instrument numbers of the antennas and preamps were not recorded. If the directions of the satellite passes were more or less evenly distributed in azimuth, then the error associated with the receiver delay shifts only the calculated elevation. This error will not, however, be reflected in the calculated precision of the coordinates. A large, short-term drift in the oscillator will also affect the coordinates (Schenke, 1982). By comparing the receiver numbers in Table 13 with the anomalously large vertical velocities, it appears that at least one receiver has a significant vertical anomaly associated with it. This receiver (number 107) shows the opposite effect on the sign of the vertical anomaly each year because the 1980 coordinates were subtracted from the 1981 coordinates to give the change in postion.

The amount that the ice sheet is thinning (or thickening) should affect all the stations in a cluster by nearly the same amount because the area of a cluster with respect to the ice sheet is small. The slopes calculated from the station elevations vary from 0.25 % to 0.8 %. Because the topography is nearly flat, the actual slope at a station probably varies by no more than 100%. Table 13 lists the expected amount of vertical motion due to downslope movement. This was calculated by multiplying the horizontal velocity at each station by the average slope for each cluster. Because average rather than point slopes were used, the values given for this movement may be in error by a multiple of two or three. Another factor that may have affected the vertical velocities was already discussed - the rotations of the coordinate system caused by fixing only five coordinates. Its effect should vary nearly linearly across a cluster.

A possible cause of error in the elevations is the inaccuracies of the meteorological data. Because the meteorological data was extrapolated from the DYE-3 site (some three kilometers from station 3003), there could be significant errors in the assumed meteorological data at any given station. If these errors are not sufficiently small (less than 5mb, less than 5 degrees Celsius and less than 5 percent in relative humidity), or if the errors do not average out, then the derived coordinates, especially elevation, will have been significanty affected. Extrapolating meteorological data over distances of more than 100 km, particularly over the ice crest, makes this source of error probable. However, this again would have affected all cluster stations that were in operation simultaneously the same way.

Nevertheless, if the following assumptions are made, then a value for the vertical velocity can be found:

1. All errors associated with technical aspects of the receivers either average out or cancel out over all the stations and over the two years. This assumption is justified because the receiver which apparently caused the largest elevation shifts was used both years, thus, canceling its effect.

2. The effect on the coordinate system caused by fixing only five coordinates does not significantly affect the vertical velocities of the stations on the ice sheet.

3. Any other systematic errors or blunders are either insignificant, cancel out or average out over the two years and over all the stations.

Given these assumptions, the average vertical velocity for all 22 stations (from the first set of solutions) is -0.15 $\pm$ 0.64 m/yr. From the third set of solutions (using the precise ephemeris in the point positioning mode) the average vertical velocity is -0.39$\pm$ 1.42 m/yr. Given the large standard deviations, these two velocities agree. These numbers can be compared with an average downslope vertical motion (of all 22 stations) of -0.12 m/yr, and the net accumulation rate of 0.36 to 0.53 meters of ice per year (Bow, 1983).

Other solutions. Relative solutions for each cluster. The results for this set of solutions are given in Tables 14, 15 and 16. These solutions also used the short arc method and the broadcast ephemeris. However, the data for the coastal stations GOT and SFJ were not included. Instead, one station in each cluster (1001, 2001 and 3005) was fixed, as closely as possible, to the coordinates obtained in the first set of solutions. Comparing the second solution results with the first derived solution shows that the horizontal coordinates do not differ systematically or significantly between solutions. While the differences between elevation results tend to be larger, they are not systematically either higher or lower. Horizontal velocities and strain rates calculated from the set of solutions do not differ significantly from the results found from the first results. The close agreement of this solution with those discussed previously provides a high degree of confidence in the overall results.

Precise ephemeris, point positioning results. In the third set of solutions the position of each station on the ice sheet was determined separately, using the precise ephemeris. These results are listed in Tables 17, 18 and 19. The precision of these results is worse than the earlier results. Formally, there are no correlations between the coordinates of different stations. However, there will actually be a small correlation between stations operating simultaneously, because the same passes were observed. The average difference of the latitudes and longitudes of this solution with those of the first solution is 0.2 meters. But these average differences have standard deviations of 0.5 meters in latitude and 0.6 meters in longitude. Therefore, they are not significant. All but two of these differences were less than 1 meter. The elevations, on the other hand, differed from the elevations of the first solution by greater amounts. Several differences were greater than 2 meters. The average difference was 0.5 meters. But the standard deviation of 0.8 meters makes this change also statistically insignificant. Velocities and strain rates were not calculated for this solution. It is unlikely that they would differ significantly from those found from the first solution, given the much larger standard deviations of these coordinates.

TABLE 14

Western Cluster Results - Short Arc Method, Cluster Stations

STA.	YR	LATITUDE	σ	LONGITUDE	σ	ELEVATION	σ	PASSES
		(° ′ ″)	(m)	(° ′ ″)	(m)	(m)	(m)	
1001	80	65 23 15.489	0.00	312 19 37.257	0.00	2021.99	0.00	320
	81	15.841	0.00	34.690	0.00	2021.95	0.00	347
	dif.	10.88 m		-33.16 m		-0.04		
1002	80	65 23 12.301	0.07	312 45 43.529	0.10	2163.42	0.08	264
	81	12.614	0.05	41.394	0.08	2161.97	0.06	343
	dif.	9.69 m		-27.57 m		-1.45		
1003	80	65 14 13.413	0.06	312 32 23.842	0.10	2126.06	0.07	196
	81	13.801	0.08	21.293	0.12	2126.43	0.09	173
	dif.	12.01 m		-33.11 m		0.37		
1004	80	65 14 32.194	0.08	312 7 45.424	0.12	1988.80	0.09	181
	81	32.584	0.07	42.397	0.11	1988.34	0.08	179
	dif.	12.14 m		-39.31 m		-0.46		
1005	80	65 23 25.307	0.06	311 53 25.096	0.09	1860.07	0.06	339
	81	25.775	0.05	21.762	0.08	1859.69	0.06	356
	dif.	14.48 m		-43.06 m		-0.38		
1006	80	65 32 48.679	0.15	312 6 16.566	0.21	1904.27	0.18	53
	81	49.037	0.08	13.331	0.13	1905.56	0.09	132
	dif.	11.07 m		-41.53 m		1.29		
1007	80	65 32 23.644	0.11	312 32 14.748	0.17	2025.45	0.12	101
	81	23.958	0.07	12.298	0.11	2025.54	0.08	205
	dif.	9.72 m		-31.49		0.09		

Unit Conversion: Latitude 1.0 m 0".032 <=> 0".10 = 3.10 m
Longitude 1.0 m = 0".08 <=> 0".10 = 1.35 m

TABLE 15

Central Cluster Results - Short Arc Method, Cluster Stations

STA.	YR	LATITUDE	σ	LONGITUDE	σ	ELEVATION	σ	PASSES
		(° ′ ″)	(m)	(° ′ ″)	(m)	(m)	(m)	
2001	80	65 6 29.394	0.00	314 18 48.295	0.00	2518.63	0.00	285
	81	29.528	0.00	47.652	0.00	2518.13	0.00	305
	dif.	4.18 m		-8.39 m		-0.50		
2002	80	65 4 2.077	0.05	314 42 46.654	0.09	2585.77	0.06	308
	81	2.225	0.06	46.201	0.11	2586.27	0.07	154
	dif.	4.58 m		-5.93 m		0.50		
2003	80	64 55 58.242	0.07	314 24 12.158	0.11	2565.60	0.08	139
	81	58.370	0.05	11.580	0.09	2566.93	0.06	196
	dif.	3.96 m		-7.60 m		1.33		
2004	80	64 58 53.300	0.06	313 58 25.219	0.11	2484.96	0.07	172
	81	53.440	0.05	24.443	0.09	2485.44	0.06	203
	dif.	4.31 m		-10.18 m		0.48		
2005	80	65 8 56.709	0.05	313 53 47.735	0.09	2441.92	0.06	263
	81	56.862	0.05	46.922	0.08	2442.81	0.06	372
	dif.	4.76 m		-10.59 m		0.89		
2006	80	65 17 34.623	0.07	314 9 57.749	0.12	2456.04	0.08	115
	81	34.803	0.07	57.028	0.11	2456.55	0.08	154
	dif.	5.55 m		-9.35 m		0.51		
2007	80	65 14 48.557	0.07	314 35 28.138	0.11	2529.50	0.08	125
	81	48.710	0.07	27.621	0.12	2530.39	0.08	145
	dif.	4.74 m		-6.71 m		0.89		

Unit Conversion: Latitude 1.0 m 0".032 <=> 0".10 3.10 m
Longitude 1.0 m 0".08 <=> 0".10 = 1.35 m

TABLE 16

Eastern Cluster Results - Short Arc Method, Cluster Stations

STA.	YR	LATITUDE	σ	LONGITUDE	σ	ELEVATION	σ	PASSES
		(° ′ ″)	(m)	(° ′ ″)	(m)	(m)	(m)	
3001	80	65 15 31.994	0.05	316 31 58.465	0.09	2414.23	0.06	283
	81	32.320	0.08	32 0.402	0.12	2413.76	0.09	88
	dif.	10.08 m		25.14 m		-0.47		
3002	80	65 6 22.207	0.05	316 32 4.252	0.09	2474.46	0.06	313
	81	22.549	0.08	5.807	0.13	2472.64	0.09	73
	dif.	10.60 m		20.29 m		-1.82		
3003	80	65 12 5.513	0.04	316 9 35.239	0.08	2522.20	0.05	316
	81	5.704	0.06	36.133	0.10	2521.58	0.07	127
	dif.	5.91 m		11.62 m		-0.62		
3004	80	65 1 31.672	0.04	316 9 7.072	0.07	2555.69	0.05	442
	81	31.920	0.05	7.925	0.09	2556.37	0.06	228
	dif.	7.66 m		11.17 m		0.68		
3005	80	65 7 7.285	0.00	315 48 8.801	0.00	2590.79	0.00	444
	81	7.428	0.00	8.530	0.00	2590.83	0.00	251
	dif.	4.44 m		5.85 m		0.04		
3006	80	64 55 35.548	0.06	315 49 12.484	0.11	2648.12	0.07	130
	81	35.688	0.08	12.862	0.13	2648.14	0.09	78
	dif.	4.35 m		4.97 m		0.02		
3007	80	65 0 50.914	0.06	315 21 24.424	0.11	2665.61	0.08	141
	81	50.987	0.06	24.353	0.10	2664.83	0.07	137
	dif.	2.26 m		-0.93 m		-0.78		
3008	80	64 50 59.192	0.06	315 20 48.394	0.09	2700.79	0.07	151
	81	59.289	0.07	48.345	0.12	2701.15	0.08	115
	dif.	3.01 m		-0.65 m		0.36		

Unit Conversion: Latitude 1.0 m = 0″.032 <=> 0″.10 = 3.10 m
Longitude 1.0 m = 0″.08 <=> 0″.10 = 1.35 m

TABLE 17

Western Cluster Results - Point Position Method, Precise Ephemeris

STA.	YR	LATITUDE	σ	LONGITUDE	σ	ELEVATION	σ	PASSES
		(° ′ ″)	(m)	(° ′ ″)	(m)	(m)	(m)	
1001	80	65 23 15.464	0.31	312 19 37.254	0.38	2021.60	0.34	134
	81	15.822	0.23	34.602	0.28	2021.18	0.27	193
	dif.	11.1 m		-34.2 m		-0.4		
1002	80	65 23 12.300	0.33	312 45 43.506	0.39	2163.47	0.35	107
	81	12.594	0.24	41.317	0.30	2161.18	0.28	176
	dif.	9.1 m		-28.3 m		-2.3		
1003	80	65 14 13.383	0.39	312 32 23.825	0.45	2126.29	0.43	78
	81	13.782	0.34	21.204	0.42	2125.94	0.40	89
	dif.	12.4 m		-33.8 m		-0.4		
1004	80	65 14 32.199	0.44	312 7 45.443	0.53	1987.65	0.49	74
	81	32.554	0.33	42.406	0.41	1986.67	0.39	96
	dif.	11.0 m		-39.2 m		-1.0		
1005	80	65 23 25.269	0.28	311 53 25.102	0.35	1959.93	0.30	138
	81	25.757	0.23	21.706	0.29	1859.64	0.28	185
	dif.	15.1 m		-43.9 m		-0.3		
1006	80	65 32 48.694	0.92	312 6 16.475	1.11	1903.67	0.97	20
	81	49.034	0.37	13.272	0.50	1903.82	0.44	65
	dif.	10.5 m		-41.4 m		0.2		
1007	80	65 32 23.593	0.64	312 32 13.800	0.80	2026.52	0.63	42
	81	23.940	0.30	12.180	0.39	2024.61	0.37	103
	dif.	10.8 m		-33.8 m		-1.9		

Unit Conversion: Latitude 1.0 m = 0.″032 <=> 0.″10 = 3.10 m
Longitude 1.0 m = 0.″08 <=> 0.″10 = 1.35 m

TABLE 18

Central Cluster Results - Point Position Method, Precise Ephemeris

STA.	YR	LATITUDE	σ	LONGITUDE	σ	ELEVATION	σ	PASSES
		(° ' ")	(m)	(° ' ")	(m)	(m)	(m)	
2001	80	65 6 29.381	0.31	314 18 48.326	0.38	2518.85	0.32	107
	81	29.527	0.24	47.566	0.29	2517.85	0.24	154
	dif.	4.5 m		-9.9 m		-1.0		
2002	80	65 4 2.066	0.27	314 42 47.641	0.34	2586.08	0.27	126
	81	2.233	0.32	46.210	0.39	2584.62	0.34	78
	dif.	5.2 m		-5.6 m		-1.5		
2003	80	64 55 58.236	0.46	314 24 12.141	0.58	2567.11	0.50	56
	81	58.387	0.28	11.494	0.36	2566.40	0.30	98
	dif.	4.7 m		-8.4 m		-0.7		
2004	80	64 58 53.289	0.38	313 58 25.282	0.48	2484.79	0.39	69
	81	53.446	0.29	24.348	0.37	2485.13	0.32	100
	dif.	4.9 m		-12.2 m		0.3		
2005	80	65 8 56.712	0.34	313 53 47.711	0.43	2441.75	0.36	101
	81	56.875	0.21	46.874	0.26	2442.57	0.23	190
	dif.	5.1 m		-10.9 m		0.8		
2006	80	65 17 34.585	0.49	314 9 57.711	0.60	2455.32	0.51	43
	81	34.818	0.34	57.003	0.41	2455.75	0.62	81
	dif.	7.2 m		-9.2 m		0.4		
2007	80	65 14 48.518	0.47	314 35 28.194	0.54	2529.00	0.47	44
	81	48.694	0.37	27.588	0.44	2528.87	0.38	79
	dif.	5.5 m		-7.9 m		-0.1		

Unit Conversion: Latitude 1.0 m = 0."032 <=> 0."10 = 3.10 m
Longitude 1.0 m = 0."08 <=> 0."10 = 1.35 m

TABLE 19

Eastern Cluster Results - Point Position Method, Precise Ephemeris

STA.	YR	LATITUDE	σ	LONGITUDE	σ	ELEVATION	σ	PASSES
		(° ′ ″)	(m)	(° ′ ″)	(m)	(m)	(m)	
3001	80	65 15 31.986	0.28	316 31 58.471	0.36	2412.95	0.31	125
	81	32.315	0.43	32 0.350	0.51	2413.69	0.44	49
	dif.	10.2 m		24.4 m		0.7		
3002	80	65 6 22.190	0.23	316 32 4.287	0.30	2474.71	0.26	143
	81	22.538	0.49	5.761	0.62	2471.18	0.52	39
	dif.	10.8 m		19.1 m		-3.5		
3003	80	65 12 5.499	0.23	316 9 35.229	0.30	2521.31	0.26	146
	81	5.705	0.34	36.069	0.42	2521.17	0.35	68
	dif.	6.4 m		10.9 m		-0.1		
3004	80	65 1 31.653	0.27	316 9 7.054	0.35	2552.52	0.30	207
	81	31.922	0.30	7.864	0.37	2556.27	0.31	105
	dif.	8.3 m		10.5 m		3.8		
3005	80	65 7 7.269	0.20	315 48 8.087	0.26	2589.97	0.21	208
	81	7.416	0.24	8.496	0.29	2590.24	0.24	128
	dif.	4.6 m		5.3 m		0.3		
3006	80	64 55 35.541	0.36	315 49 12.468	0.45	2647.43	0.38	65
	81	35.700	0.44	12.859	0.54	2647.68	0.44	36
	dif.	4.9 m		5.1 m		0.3		
3007	80	65 0 50.863	0.34	315 21 24.455	0.43	2666.54	0.35	70
	81	50.978	0.40	24.342	0.53	2664.31	0.42	57
	dif.	3.6 m		-1.5 m		-2.2		
3008	80	64 50 59.179	0.32	315 20 48.447	0.40	2700.24	0.34	74
	81	59.305	0.36	48.310	0.47	2700.24	0.38	58
	dif.	3.9 m		-1.8 m		0.0		

Unit Conversion: Latitude 1.0 m = 0".032 <=> 0".10 = 3.10 m
Longitude 1.0 m = 0".08 <=> 0".10 = 1.35 m

Ellipsoidal Elevations versus Mean Sea Level Elevations

All of the elevations used in this report, except for those in Figure 3, are elevations above the WGS66 ellipsoid ($a=6378145$ m, $1/f=298.25$). General practice, however, is to give elevations above mean sea level (MSL). The difference between the two elevations is called the geoid undulation. Geoid undulations were computed from 1° x 1° mean gravity anomalies and the GEM9 potential coefficients. The mean gravity anomalies were taken from the master tape maintained by the Department of Geodetic Science and Surveying, The Ohio State University. The geoid undulations were computed using the modified Molodensky's truncation method with Meissl's modification (Jekeli, 1980). The geoid undulations for all the stations are listed in Table 20. To obtain MSL elevations, the geoid undulations should be subtracted from the ellipsoidal elevations.

TABLE 20

Geoid Undulations

COASTAL STATIONS	
STA.	GEOID UNDULATION[1]
	(m)
GOT	27.9
SFJ	32.9

WESTERN CLUSTER		CENTRAL CLUSTER		EASTERN CLUSTER	
STA.	GEOID UNDULATION[1]	STA.	GEOID UNDULATION[1]	STA.	GEIOD UNDULATION[1]
	(m)		(m)		(m)
1001	38.8	2001	42.1	3001	45.8
1002	39.6	2002	42.8	3002	45.9
1003	39.1	2003	42.2	3003	45.2
1004	38.3	2004	41.5	3004	45.2
1005	37.8	2005	41.4	3005	44.6
1006	38.5	2006	41.9	3006	44.6
1007	39.3	2007	42.5	3007	43.8
				3008	43.8

[1] Geoid Undulation - The difference between an elevation above the ellipsoid and the geoid. To obtain an elevation above mean sea level subtract the geoid undulation from the elevation above the ellipsoid. All elevations listed in this report are above the WGS66 ellipsoid.

SUMMARY AND CONCLUSIONS

The locations determined for the 22 stations on the ice sheet of Greenland were found with average (RMS) standard deviations of 0.066 meters in latitude, 0.105 meters in longitude and 0.096 meters in elevation. These are the formal errors as calculated in the program GEODOP. The accuracy of these positions also may be affected by changes in the coordinate system, errors in the atmospheric corrections, receiver timing delay errors and errors in determining the antenna offsets. The first source of error occurred because only two receivers were placed at permanent, nonmoving sites instead of three. These stations were used to provide a fixed reference so that the short arc method could be used to improve the relative accuracy of the locations of the moving stations. With only two permanent stations, only five coordinates could be fixed, thus, one rotation was still possible. However, the result of this rotation on the coordinates of the stations on the ice sheet appears to be insignificant. If there is any rotation because of the geometry of the stations, the principal effect will be in the elevations of the stations on the ice sheet.

The atmospheric correction also principally affects elevations. Because the meteorological data had to be extrapolated from one location to all the stations on the ice sheet, the errors due to this may be significant. This error source should affect all stations operating simultaneously within a cluster in a similar manner. This error source does not appear to be a particular problem in this analysis.

Another problem is that the antenna positions the second year may not have been correctly determined relative to their positions in the first year. All of the horizontal offsets except one appear to be less than two or three centimeters. One station (1005) may have been displaced horizontally up to ten centimeters. The vertical offsets were determined relative to datums placed in the ice near the station. Unfortunately, there is reason to believe that there may be uncorrected blunders in the data recorded. It is also possible that either the receiver delays were not the same for each receiver, as had been implicitly assumed, or that there were large short-term drifts in the oscillator frequencies. An error in either of these will force the calculated position of the station to shift. Because the passes were well distributed in azimuth and elevation, if this error occurred, it would have principally affected the elevations. Most of the other vertical velocities that appear unreasonable occurred at stations where one specific MX1502 receiver was used (No. 107). It is assumed that either the calibration of this receiver was in error or that the oscillator was unstable. The reason for the remaining unreasonable velocities is unknown.

A general problem with determining velocities and strain rates is determining to which date the calculated coordinates refer. Because all the stations in a cluster were calculated simultaneously, their coordinates all refer to the same date. It was determined that the median date of the total span of observations at a given cluster could be used with acceptable accuracy as the date to which the coordinates refer. If a date other than the median date is used, however, the error would be canceled or nearly canceled when the time span between the occupations was determined. Thus, it is not felt that the error in dating the coordinates could be more than several days, so that its effect on either the velocities or the strain rates would not be more than 1 percent.

Figure 8 shows the velocities at all the clusters. The contours were determined from the calculated elevations for 1980. These elevations are above the WGS66 ellipsoid, rather than above sea level. This plot shows that the ice is moving in, or close to, the direction of maximum slope. Also seen is the increase of velocity with increasing slope. This is in accordance with theory (Paterson, 1969).

The velocity of station 3003 in the Eastern Cluster was found to be 13.44 ± 0.14 m/yr at an azimuth of 63.8 ± 0.5 degrees. An earlier velocity for a site about three kilometers south of this station was 12.7 m/yr at an azimuth of 61 degrees (Mock, 1976). The accuracy of this velocity is not given, but it must be at least $\pm$ 1.0 m/yr because it was determined from positions which were determined from only 30 to 40 NNSS satellite passes using the precise ephemeris. Thus, the two velocities agree

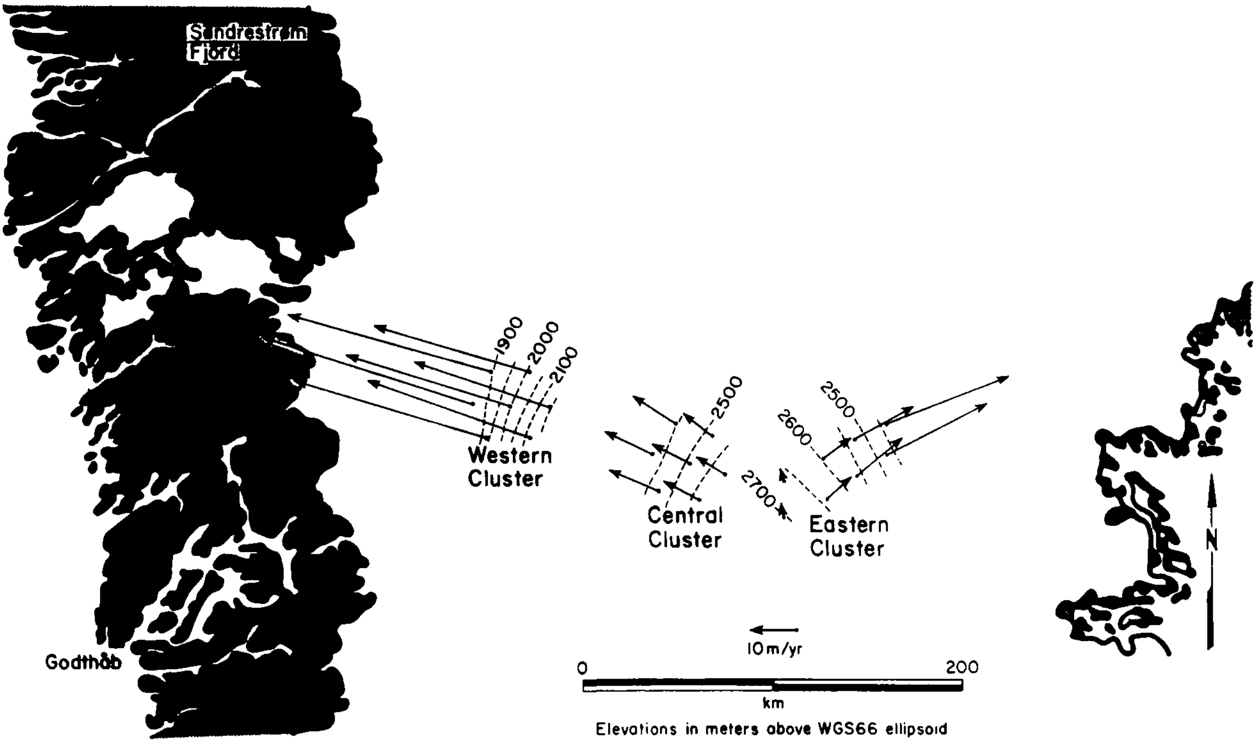


Illustration 8: Greenland Ice Sheet Station Velocities

For elevations above mean sea level see p. 56.

well.[1]

From our experiences during two field seasons in Greenland, we learned several lessons that would modify our activities when we conduct similar experiments elsewhere. For exanple the receiver/antenna/preamp combinations should be kept the same and the combinations should be calibrated both before and after their use in the field. This may be done by making observations of all the receivers at the same location. It would be preferable if the earth-centered coordinates of this position were known.

Detailed records should be kept of each visit to each receiver. For MX1502's, the information recorded should include date, time, error messages, tape used and passes recorded. The information recorded will vary with the receiver type. Each cassette should be used for only one station even if it is not completely filled. However, if a cassette must be used for more than one series of passes, it must not be used at the same station. This avoids unreasonable effort in inputting the data.

When a receiver is removed from a site, the point position of the station calculated by the receiver should be recorded along with any other information on the station location and setup.

Accurate meteorological data should be kept, preferably from every receiver station. Self-recording instruments would be a definite advantage as the data should be recorded three or four times a day, at a minimum. These instruments should be calibrated both before and after their use in the field.

----

[1] As part of another experiment carried out by GISP during both the 1980 and the 1981 field seasons, the positions of station 3003 and Mock's Geoceiver station were linked in a conventional survey (Whillans, and others, 1983). From the results of that experiment, Mock's station is moving 0.08 m/yr slower than station 3003 and at a relative azimuth of -1.0 degrees (relative to the motion of station 3003). This also agrees with the results presented in this report.

With 40 to 50 jointly observed passes, the relative positions of the observing stations may be obtained to within 0.20 meters using the short arc method. However, 10 to 20 percent more passes should be observed at each station to be assured of having enough usable passes. Using the precise ephemeris and the point positioning method, 40 to 50 usable passes will give station accuracies of under one meter. To fully fix the coordinate system, receivers must be placed at three well distributed, nonmoving sites. Additionally, these sites must be close enough to allow a significant fraction of the passes observed also to be observed at the stations whose coordinates are being sought. To allow good symmetry they should not be too close to each other. If this cannot be done, consideration should be given to using the point positioning method with the precise ephemeris.

References

Archinal, B. A. (1982). A Comparison of Geodetic Doppler Satellite Receivers, MS Thesis, Ohio State University, Dept. of Geodetic Science and Surveying and Report 340 of the Dept. of Geodetic Science and Surveying, 105 pp.

Bow, W. A. (1983). Isotropic Diffusion in Polar Firn, Institute of Polar Studies Report 81, The Ohio State University, 60 pp.

Brown, Duane C. (1976) "Doppler Positioning by the Short Arc Method," Proceedings of the International Symposium on Satellite Doppler Positioning, Physical Sciences Laboratory, New Mexico State University, Las Cruces, New Mexico, 88003, pp. 97-140.

Hofmann, W. (1974) "Die Internationale Glaziologische Gronland - Expedition (EGIG)", 2. "Die geodatische Lagemessung - Eisbewegung 195967 in den EGIG-Profilen." Zeitschrift fur Gletscherkunde und Glazialgeologie, X, pp. 217-224.

Jekeli, C. (1980). Reducing the Error of Geoid Undulation Computations by Modifying Stokes' Function, Report 301 of the Dept. of Geodetic Science and Surveying. The Ohio State University, 50 pp.

Karsten, A. and M. Stober (1975). "Deformation-messungen auf dem Gronlandeschen Inlandeis wahrend der Intern. Glaziologischen Gronland expedition 1974", Polarforschung, 45, 1, pp. 45-50.

Kouba, J. and Boal, J.D. (1976). Program GEODOP, Dept. of Energy, Mines and Resources, Ottawa, Ontario, 143 pp.

Livieratos, E., (1980). "Crustal Strains Using Geodetic Methods.", Quaterniones Geodaesiae, 3, pp. 191-211.

Malzer, H. (1968) "Das Nievellement uber das gronlandische Inlandeise der Internationalen Glaziologischen Gronland expedition (EGIG) 1959", Meddelelser on Gronland, 173, 7.

Mock, S. J. (1976), "Geodetic Postions of boreholes sites of the Greenland Ice Sheet Program." CRREL Report 76-41. Hanover: U.S. Army Cold Regions Research and Engineering Laboratory, 7 pp.

Paterson, W.S.B. (1969). The Physics of Glaciers in Pergamon Press, NY. Chapters 5 and 6.

Paterson, W.S.B. (1980). "Ice Sheets and Ice Shelves." Dynamics of Snow and Ice Masses ed. S.C. Colbeck, Academic Press, Inc. NY.

Rapp, R.H. (1979). "Geometric Geodesy Volume I (Basic Principles)", Class notes. Dept. of Geodetic Science and Surveying, The Ohio State Univ.

Schenke, H. W. (1982). "Precision Doppler Satellite Observation in the German Three-Dimensional Testnet 'Western Harz'", Proceedings of the Third International Symposium on Satellite Doppler Positioning. 1. pp. 291-315.

Seckel, H. and M. Stober (1968) "Hohenanderung des gronlandischen Inlandeises, 1959-1968." Polarforschung, VI, 1/2, pp. 215-221.

Stober, M. (1976), "zur Bestimmung der Deformation des gronlandischen Inlandeis bei Crete", Polarforschung, 46, 1, pp. 60-61.

Weidick, A. (1977). A Review of Quaternary Investigations in Greenland. Institute of Polar Studies, Rept. No. 55, The Ohio State University, 161 pp.

Whillans, I. M., K. C. Jezek, and A. J. R. Drew, and N. Gundestrup (1983). "Ice Flow Near the GISP Borehole." Paper presented at the Symposium on Ice and Climate Modelling, Northwestern University, Evanston, Ill. June 1983.